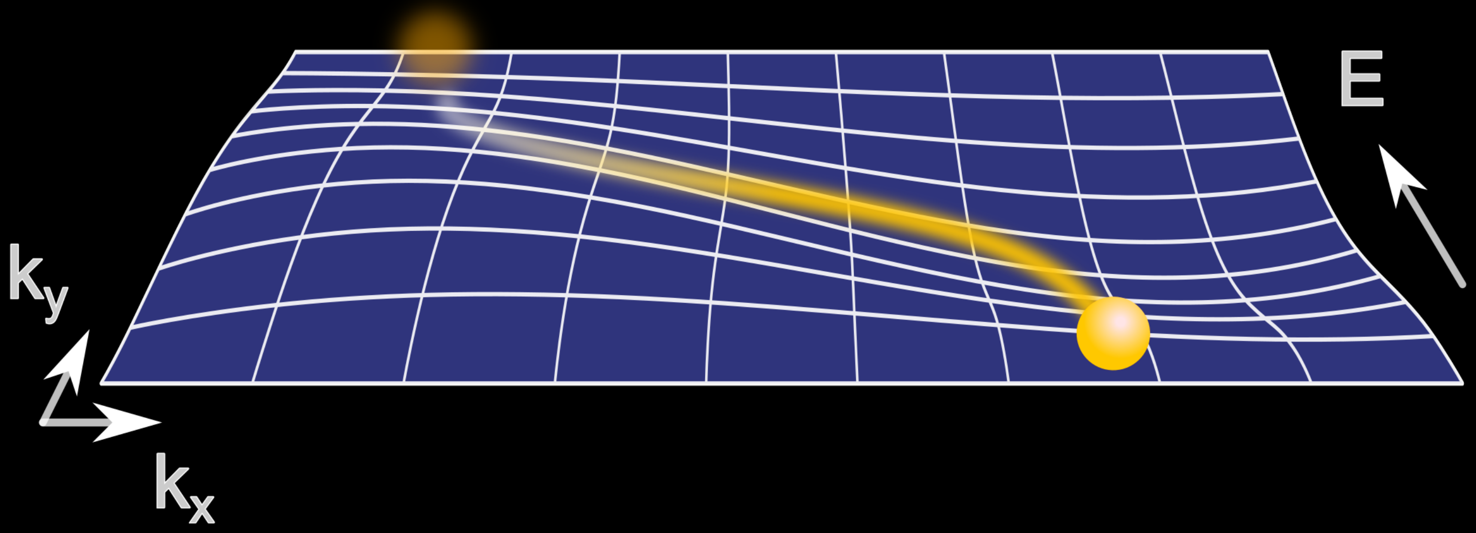
The Quantum Geometry of 2D Electron Gases

Giacomo Sala



Université de Genève

Binasuan

(Philippine Wine Glass Dance)

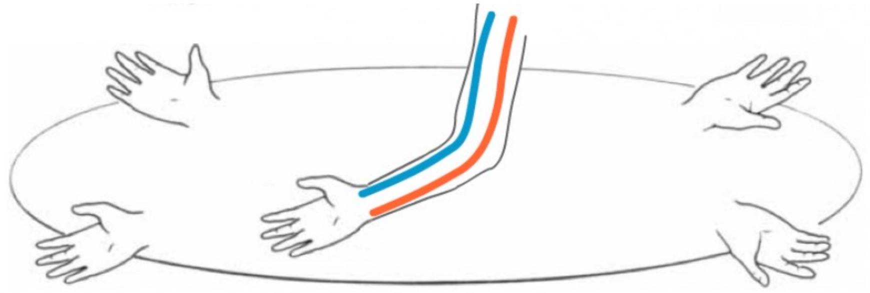
BINASUAN

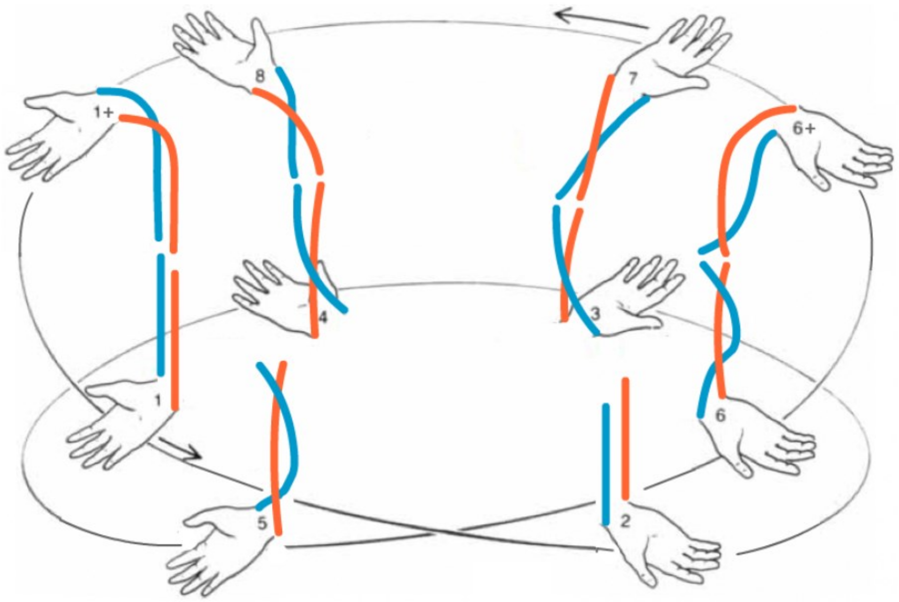
(PHILIPPINE WINE GLASS DANCE)

https://www.youtube.com/watch?v=o5RN2P-0D3w

BINASUAN

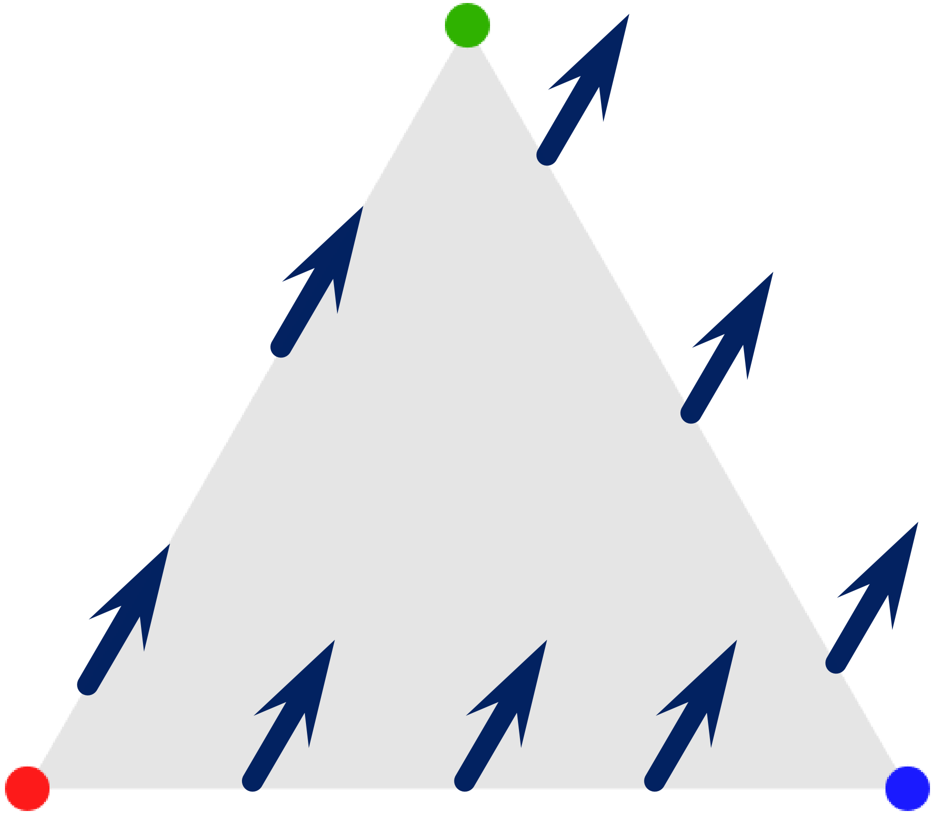
(PHILIPPINE WINE GLASS DANCE)



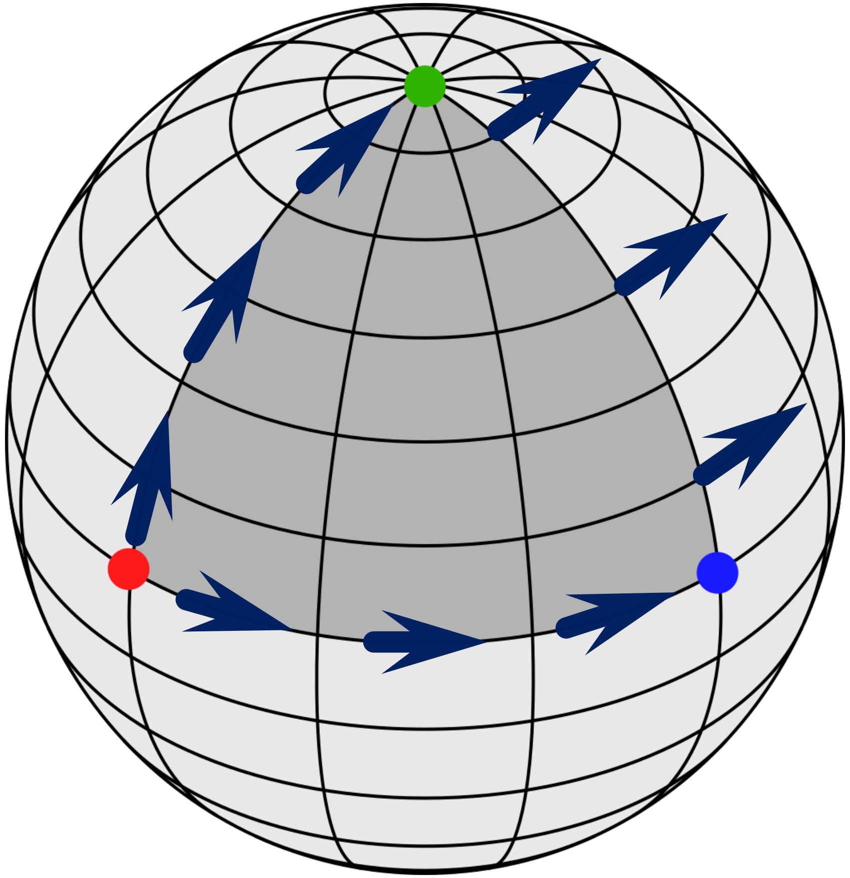


https://www.math.stonybrook.edu/~tony/fb-and-qt/Sci-Am-errata.pdf

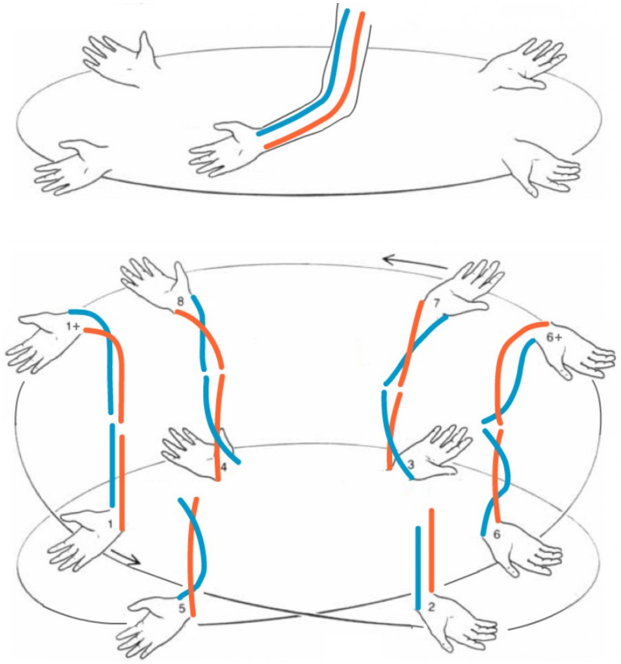
Parallel Transport

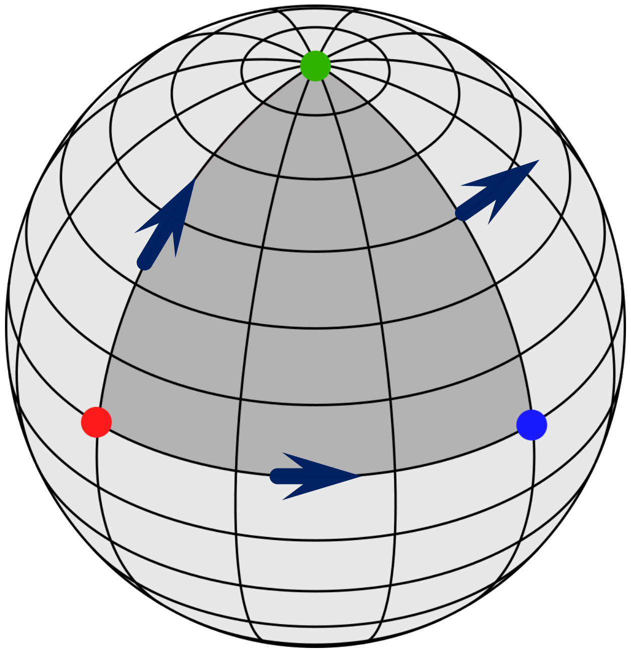


Parallel Transport



Curved Spaces





OUTLINE

- ❑ Quantum geometry
- ❑ Spin-momentum locking
- ❑ Linear and nonlinear magnetotransport

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- ❑ Quantum geometry
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The Origin of Quantum Geometry

$$H(\mathbf{k}) = \mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

$$E_{\pm}(\mathbf{k}) = \pm h(\mathbf{k})$$

$$|\psi_{\pm}(\mathbf{k})\rangle \sim \begin{pmatrix} \pm h + h_z \\ h_x + i h_y \end{pmatrix}$$

The Origin of Quantum Geometry

$$|\psi(\mathbf{k})\rangle = \begin{pmatrix} \alpha + i\beta \\ \gamma + i\delta \end{pmatrix}$$

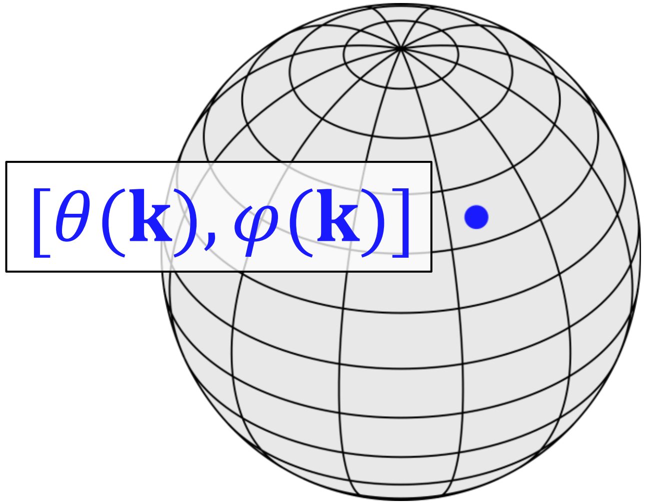
Phase independence

Normalization

$$\delta = 0$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

$$|\psi(\mathbf{k})\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) e^{-i\varphi} \\ \sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$



THE ORIGIN OF QUANTUM GEOMETRY

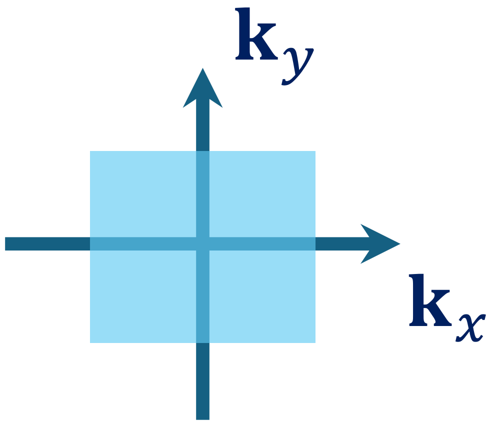
Phase independence

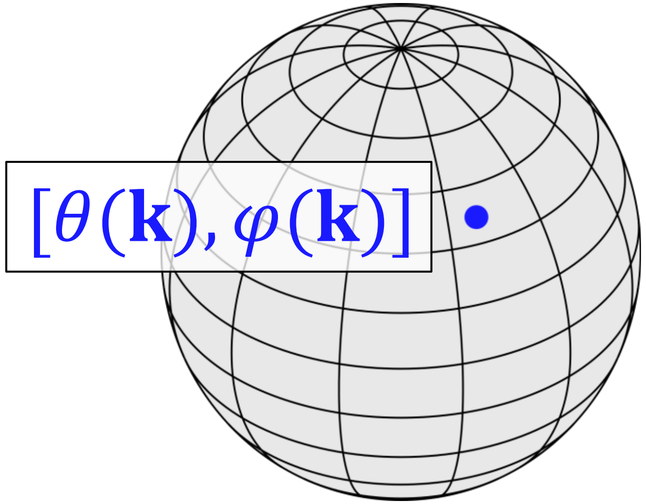
$$|\psi(\mathbf{k})\rangle = \begin{pmatrix} \alpha + i\beta \\ \gamma + i\delta \end{pmatrix}$$

$$\delta = 0$$

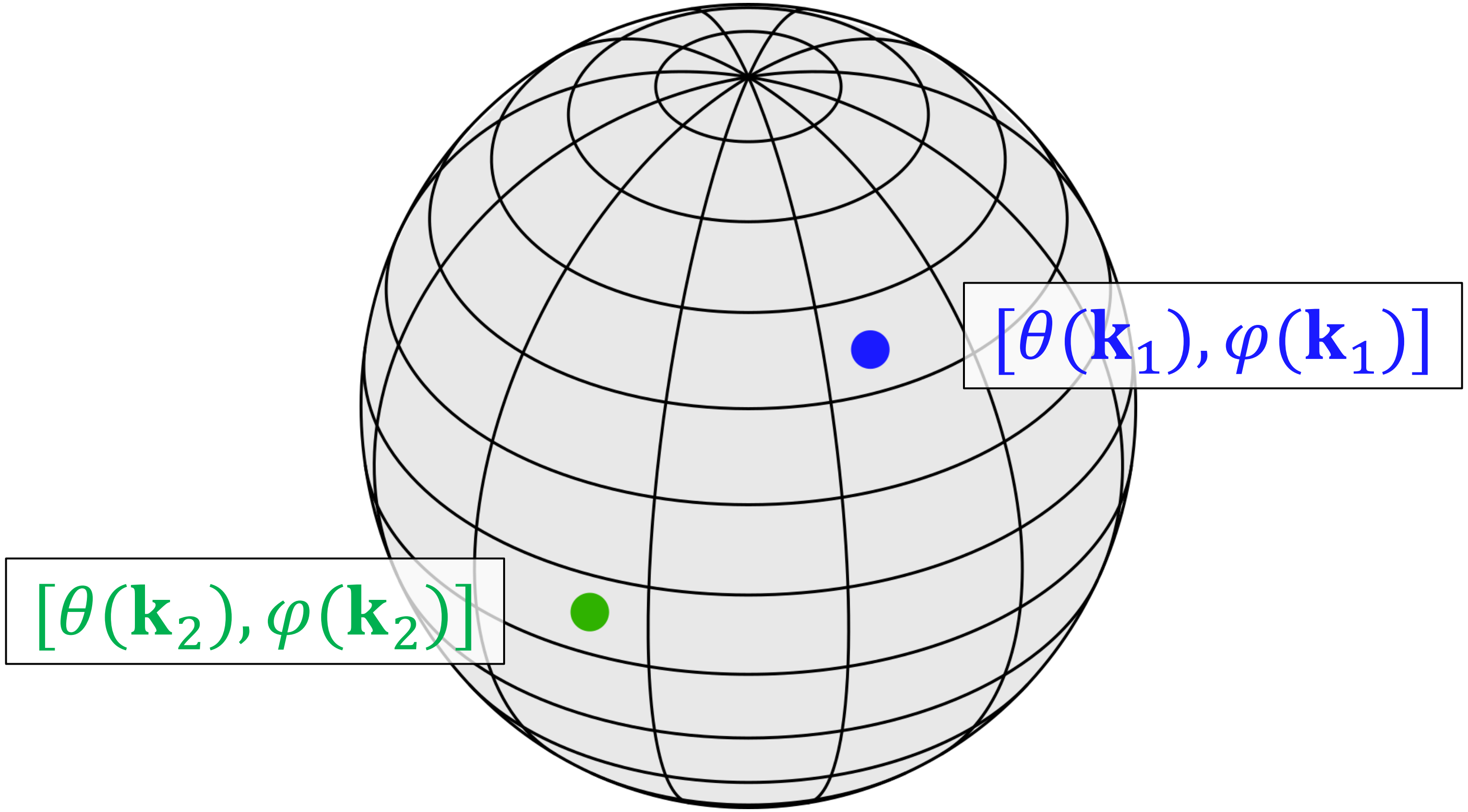
$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

Normalization

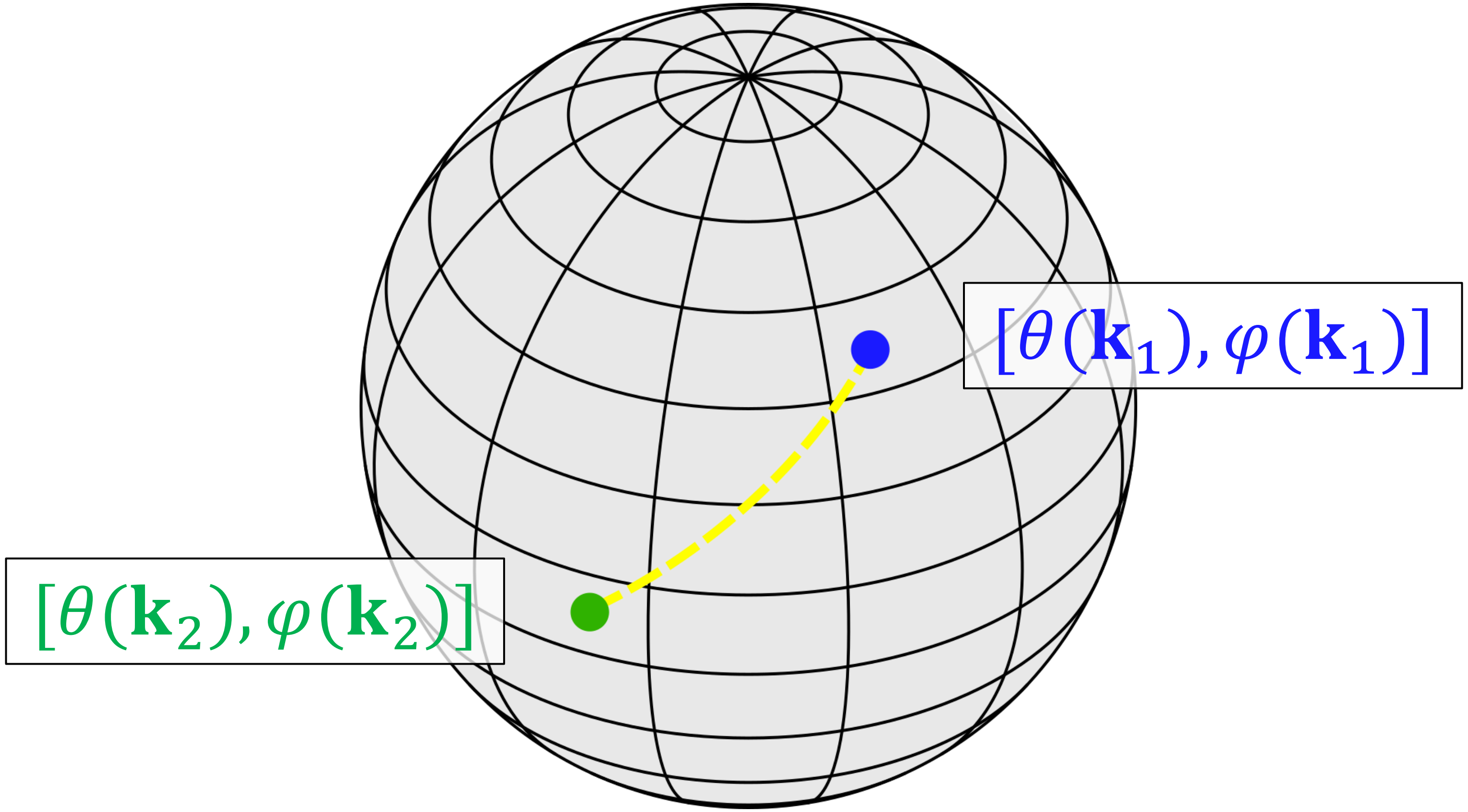




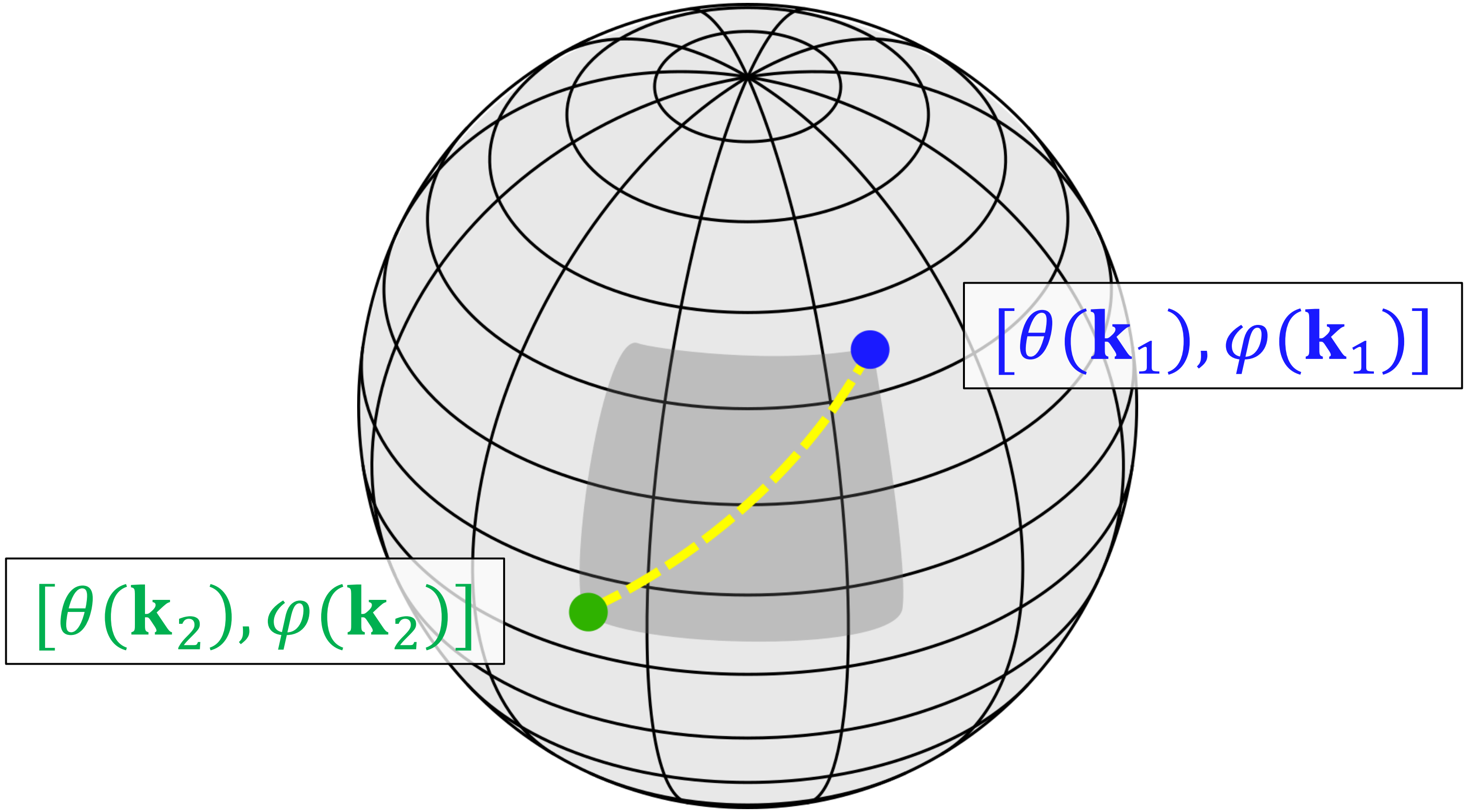
The Origin of Quantum Geometry



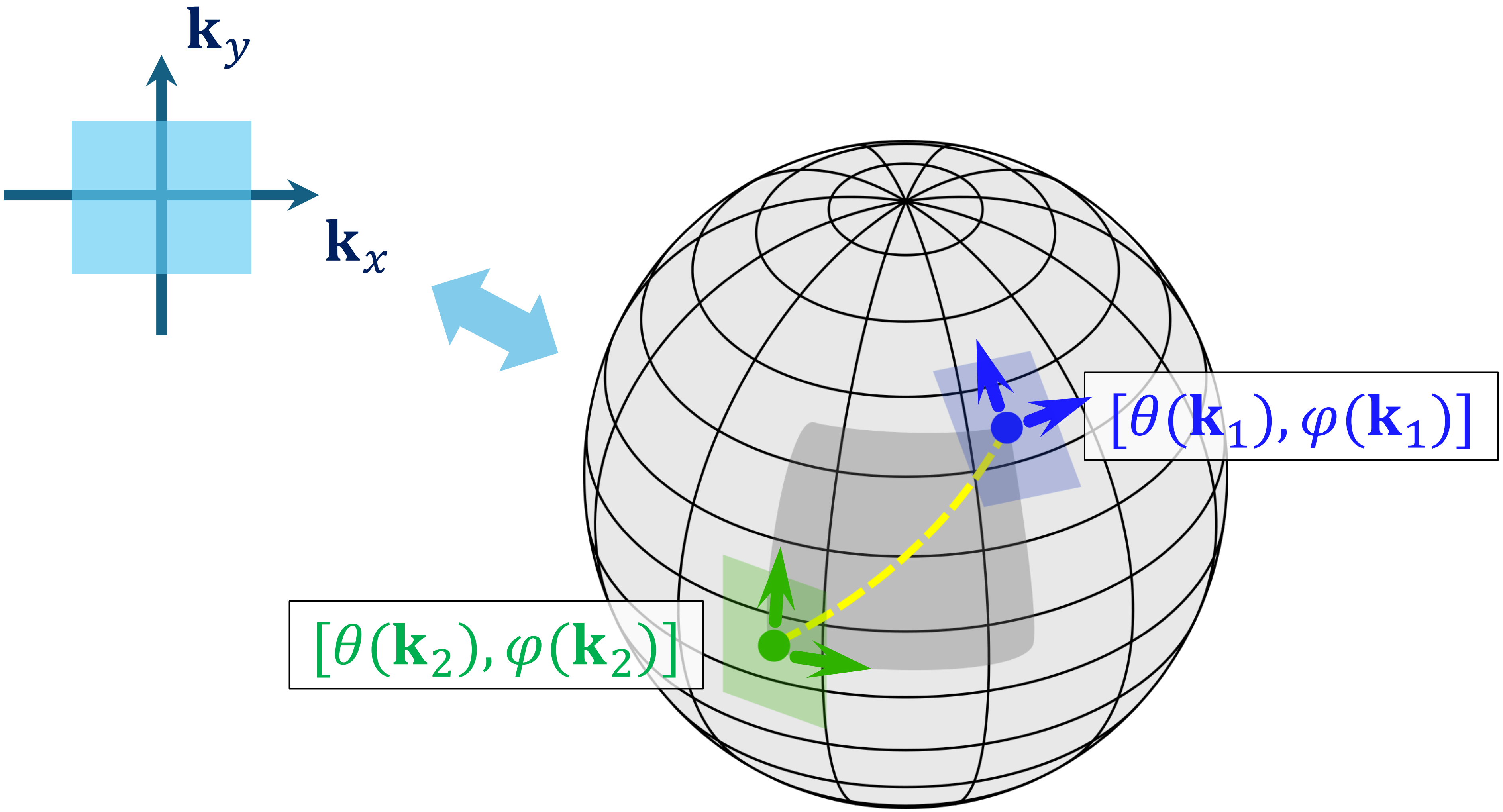
The Origin of Quantum Geometry



The Origin of Quantum Geometry



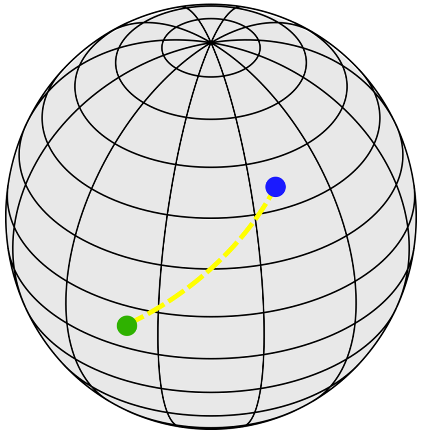
The Origin of Quantum Geometry



THE QUANTUM GEOMETRY

Quantum metric

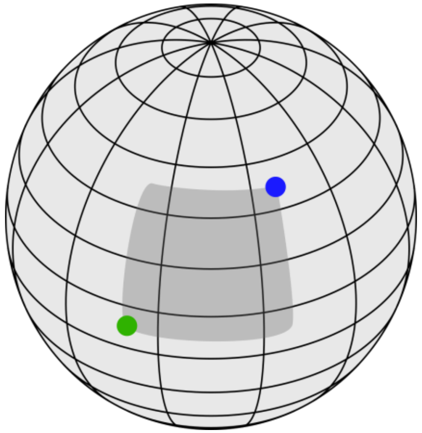
g_{ab}



$$ds^2 = g_{ab} dx_a dx_b$$

Berry curvature

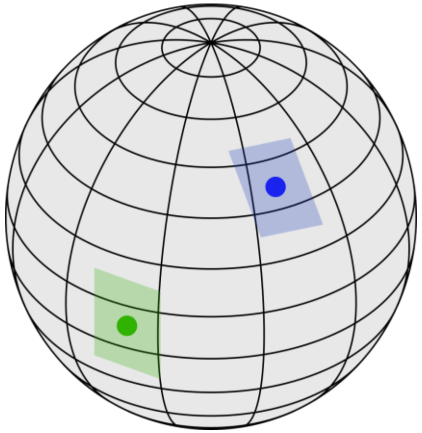
Ω_{ab}



$$dS = \Omega_{ab}\, dx_a^1 \wedge dx_b^2$$

Geometric connection

Q_{abc}

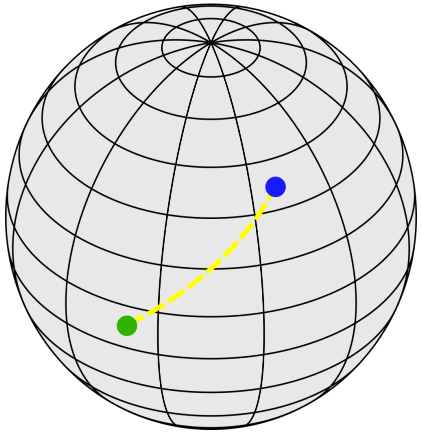


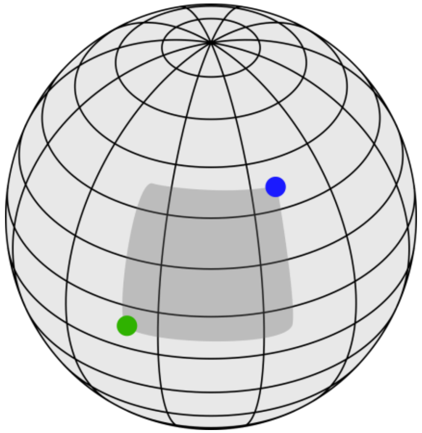
THE QUANTUM GEOMETRY

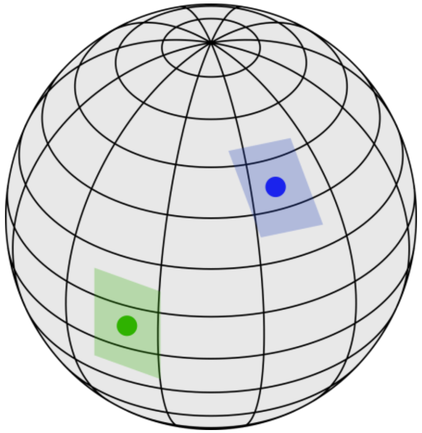
Quantum metric g_{ab}

Berry curvature Ω_{ab}

Geometric connection Q_{abc}







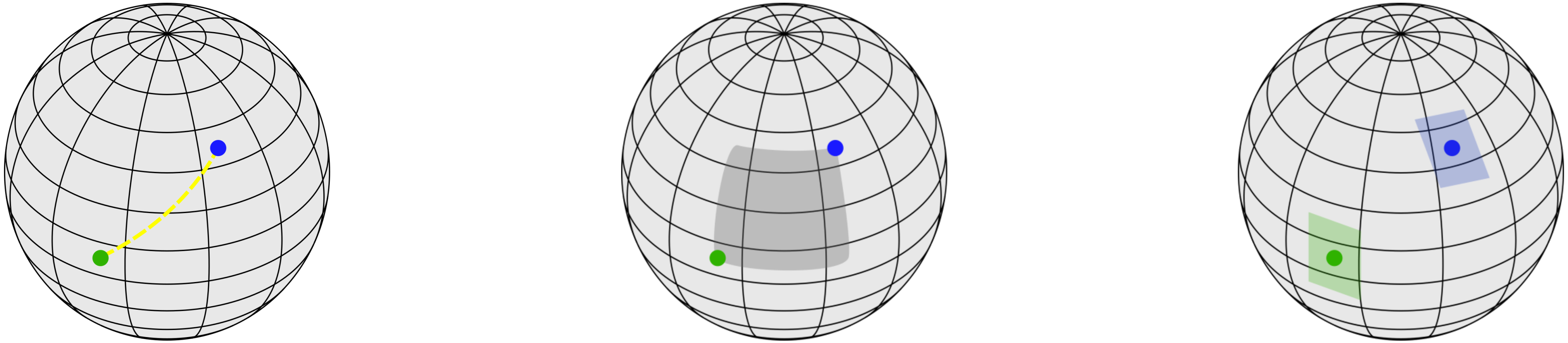
$$ds^2 = d\theta^2 + (\sin\theta)^2\, d\varphi^2 \qquad dS = \sin\theta\, d\theta d\varphi$$

$$g_{ab} = \begin{bmatrix} 1 & 0 \\ 0 & (\sin\theta)^2 \end{bmatrix} \qquad \Omega_{ab} = \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$$

The Quantum Geometry

Quantum metric | Berry curvature | Geometric connection

g_{ab} | Ω_{ab} | Q_{abc}



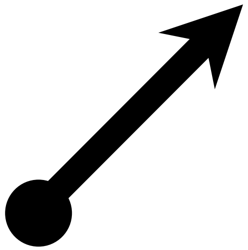
$$H(\mathbf{k}) = \mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

$$g_{ab} = \frac{1}{4} \partial_{k^a} \hat{\mathbf{h}} \cdot \partial_{k^b} \hat{\mathbf{h}}$$

$$\Omega_{ab} = \frac{\hat{\mathbf{h}}}{2} \cdot \left(\partial_{k^a} \hat{\mathbf{h}} \times \partial_{k^b} \hat{\mathbf{h}} \right)$$

The Quantum Geometry

Quantum metric	Berry curvature	Geometric connection
g_{ab}	Ω_{ab}	Q_{abc}



$$\boldsymbol{v} = [a, b, c]$$

$$l^2 = \boldsymbol{v}^* \cdot \boldsymbol{v}$$

The Quantum Geometry

Quantum metric

g_{ab}

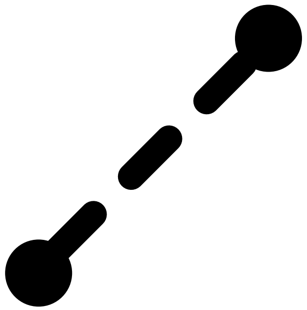
Berry curvature

Ω_{ab}

Geometric connection

Q_{abc}

$|\Psi(\mathbf{k}+d\mathbf{k})\rangle$



$|\Psi(\mathbf{k})\rangle$

$$ds^2 = 1 - |\langle\Psi(\mathbf{k}+d\mathbf{k})|\Psi(\mathbf{k})\rangle|^2$$

$$= \sum_{ij} g_{ij} d\mathbf{k}_i d\mathbf{k}_j$$

Why the Quantum Geometry?

Quantum metric

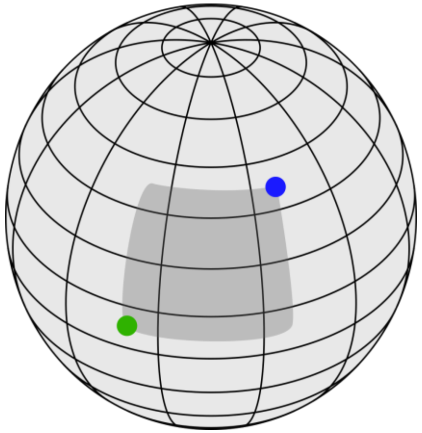
g_{ab}

Berry curvature

Ω_{ab}

Geometric connection

Q_{abc}



$$\varphi_{\mathrm{B}} = \int_{S} \boldsymbol{\Omega} \cdot d\mathbf{S}$$

WHY THE QUANTUM GEOMETRY?

Quantum metric

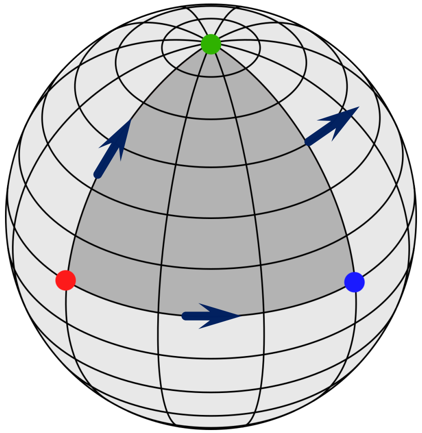
g_{ab}

Berry curvature

Ω_{ab}

Geometric connection

Q_{abc}



$$\varphi_{\mathrm{B}} = \int_{S} \mathbf{\Omega} \cdot d\mathbf{S}$$

WHY THE QUANTUM GEOMETRY?

Quantum metric	Berry curvature	Geometric connection
g_{ab}	Ω_{ab}	Q_{abc}

- Berry phase
- Anomalous Hall effect(s)
- Quantum Hall effect
- Topological states of matter
- Orbital magnetism

Berry, Geometric phases in physics 5, 1989
Chang, J. Phys. Condens. Matter 20, 193202, 2008
Nagaosa, Rev. Mod. Phys. 82, 1539, 2010
Hasan, Rev. Mod. Phys. 82, 3045, 2010

WHY THE QUANTUM GEOMETRY?

Quantum metric	Berry curvature	Geometric connection
g_{ab}	Ω_{ab}	Q_{abc}

$$G_{ab} = g_{ab} - \frac{i}{2}\Omega_{ab}$$

$$\mathrm{Tr}[\mathbf{g}] \geq \boldsymbol{\Omega}$$

$$\mathrm{Det}[\mathbf{g}] \geq \frac{\boldsymbol{\Omega}^2}{4}$$

Rossi, Curr. O. S. S. Mat. Sci. 25, 100952, 2021
Torma, Phys. Rev. Lett. 131, 240001, 2023
Liu, Nat. Sc. Rev. 12: nwae334, 2025
Verma, arXiv2504.07173v1

WHY THE QUANTUM GEOMETRY?

Quantum fidelity

Zanardi, PRL 99, 100603, 2007
Garnerone, PRA 79, 032302, 2009

Orbital magnetism

Gao, PRB 91, 214405, 2015
Piechon, PRB 94, 134423, 2016
Freimuth, PRB 95, 184428, 2017

Optical transitions

Ahn, PRX 10, 041041, 2020
Ahn, Nat. Phys. 18, 290, 2022
Ma, Nat. Rev. Phys. 5, 170, 2023
Komissarov, Nat. Comm. 15, 4621, 2024

Nonlinear electronic transport

Das, PRB 108, L201405, 2024
Kaplan, PRL 132, 026301, 2024
Mandal, PRB 110, 195131, 2024
Liu, Nat. Sc. Rev. 12: nwae334, 2025
Jiang, arXiv2503.04943v1

Nonlinear valley Hall effect

Das, PRL 132, 096302, 2024

Flat-band superconductivity

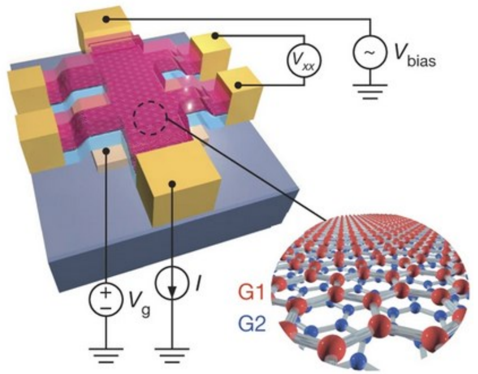
Peotta, Nat. Comm. 6, 8944, 2015
Huhtinen, PRB 106, 014518, 2022
Tian, Nat. 614, 440, 2023
Yu, arXiv2501.00098v1

Electron-phonon coupling

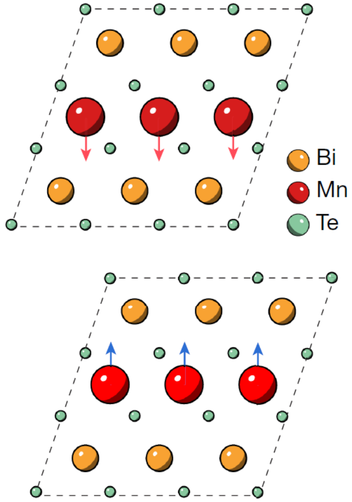
Yu, Nat. Phys. 20, 1262, 2024

Which Materials?

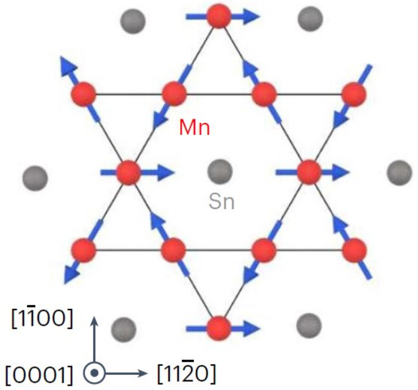
MATBG



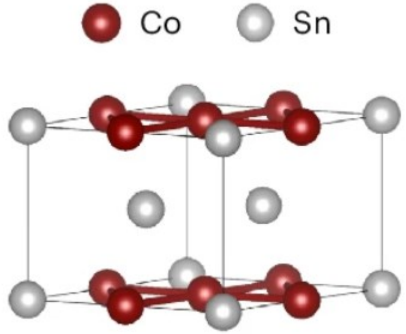
$MnBi_2Te_4$



Mn_3Sn



CoSn



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Gao, Science 381, 181, 2023
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Kang, Nat. Phys. 24, 2024

OUTLINE

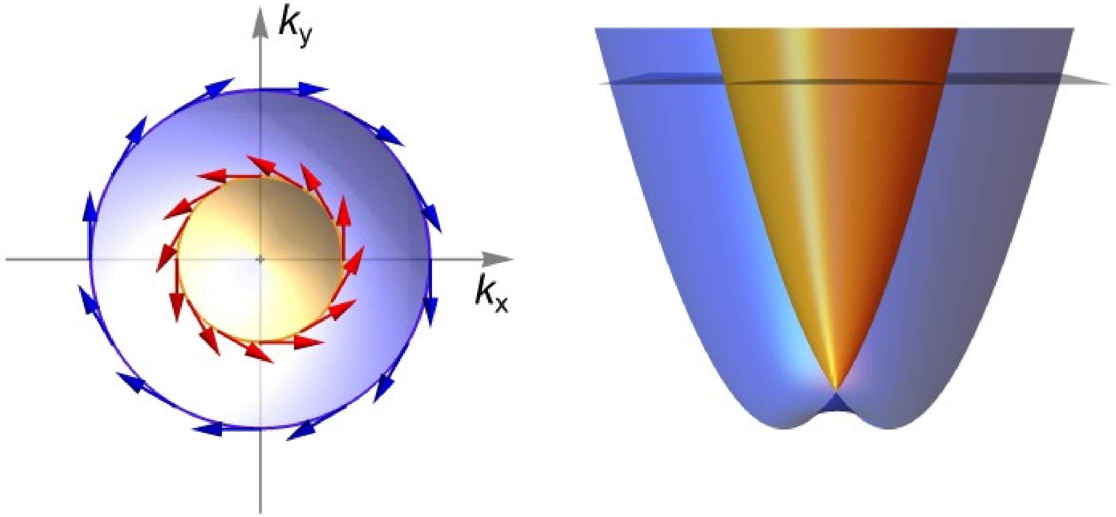
- ❑ Quantum geometry
- ❑ Spin-momentum locking
- ❑ Linear and nonlinear magnetotransport

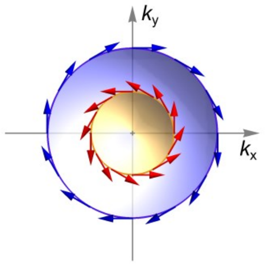
Spin-Momentum Locking

- Metallic surface states
- 2D electron gases
- Bulk Rashba semiconductors
- Topological surface states

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + \alpha(\mathbf{k} \times \mathbf{z}) \cdot \boldsymbol{\sigma}$$

$$= h_0(k)\mathbb{I} + \mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$





Spin-Momentum Locking

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + \alpha(\mathbf{k} \times \mathbf{z}) \cdot \boldsymbol{\sigma}$$
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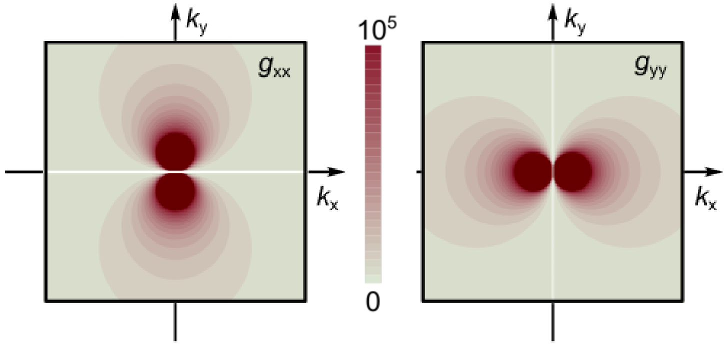
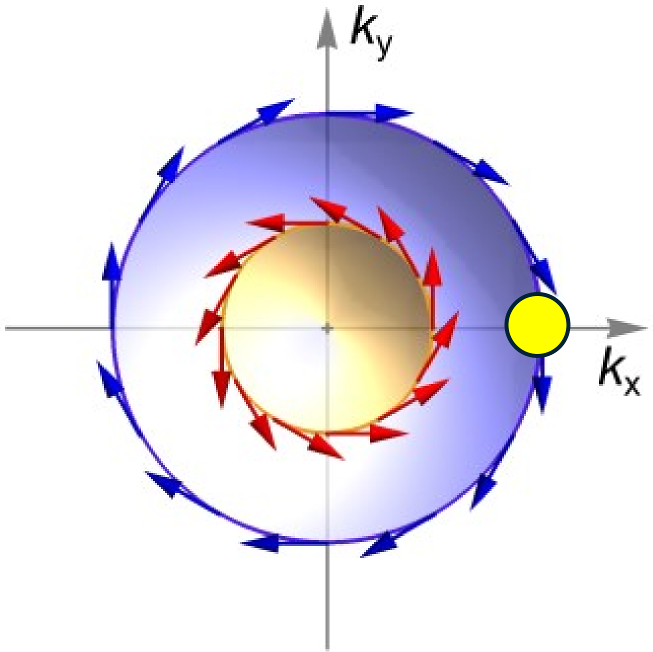
Quantum metric

$$g_{ab} = \frac{1}{4} \partial_{k^a} \hat{\mathbf{h}} \cdot \partial_{k^b} \hat{\mathbf{h}}$$



Spin-Momentum Locking

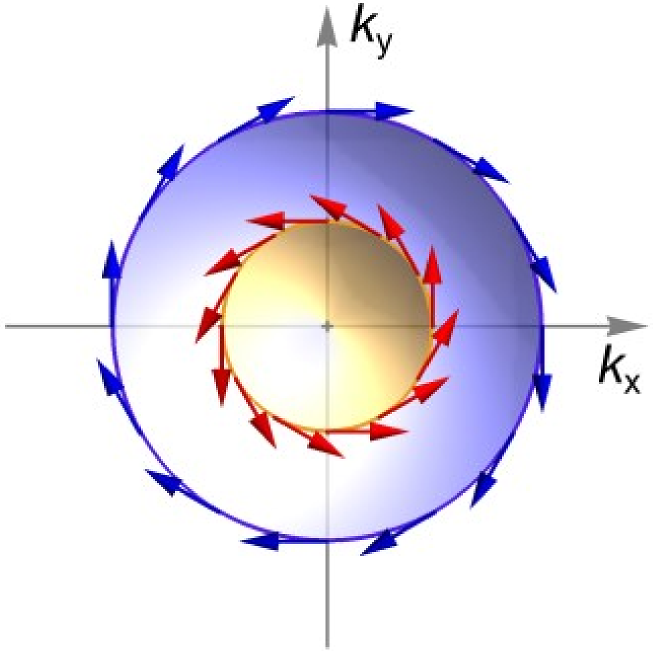


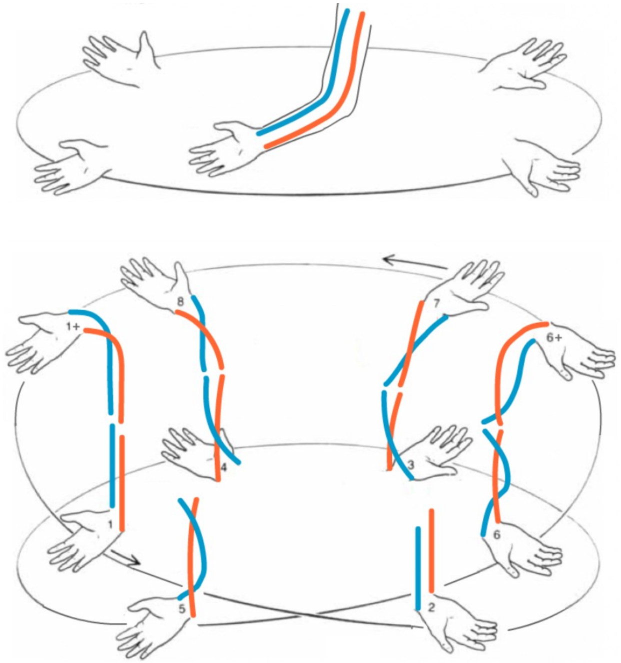
$$|\psi(\varphi,\theta)\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) e^{i\frac{\varphi}{2}} \\ \sin\left(\frac{\theta}{2}\right) e^{-i\frac{\varphi}{2}} \end{pmatrix}$$

$$\theta = \frac{\pi}{2}$$

$$\left|\psi\left(0,\frac{\pi}{2}\right)\right\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \Rightarrow \quad \left|\psi\left(0+2\pi,\frac{\pi}{2}\right)\right\rangle = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Spin-Momentum Locking





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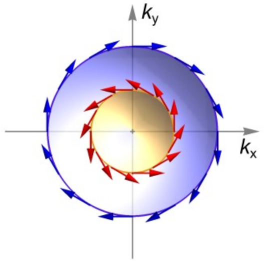
Spin-Momentum Locking



$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + \alpha(\mathbf{k} \times \mathbf{z}) \cdot \boldsymbol{\sigma}$$

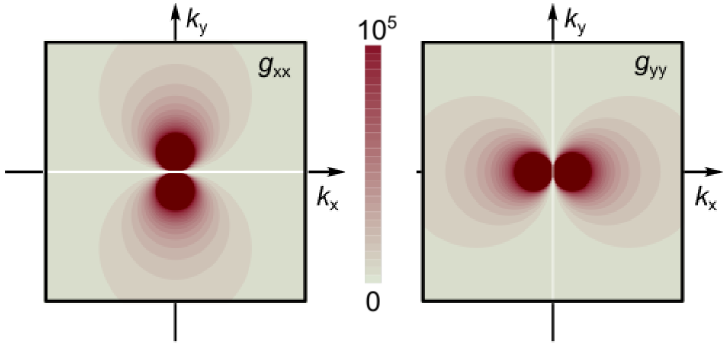
$$= h_0(k)\mathbb{I} + \mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

Quantum metric

$$g_{ab} = \frac{1}{4}\partial_{k^a}\hat{\mathbf{h}} \cdot \partial_{k^b}\hat{\mathbf{h}}$$

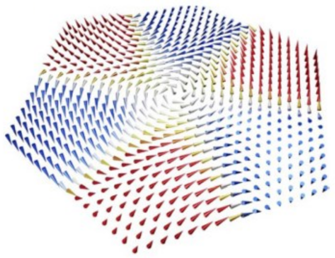
Berry curvature

$$\Omega_{ab} = \frac{\hat{\mathbf{h}}}{2} \cdot \left(\partial_{k^a}\hat{\mathbf{h}} \times \partial_{k^b}\hat{\mathbf{h}}\right)$$



$$\delta(\mathbf{k})$$

Spin-Momentum Locking



Lesne, Nat. Mater. 22, 576, 2023

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + \alpha(\mathbf{k} \times \mathbf{z}) \cdot \boldsymbol{\sigma} + \frac{\lambda}{2}(k_+^3 + k_-^3)\sigma_z$$
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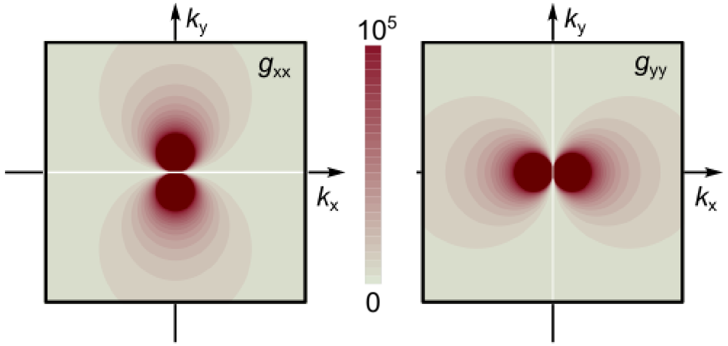
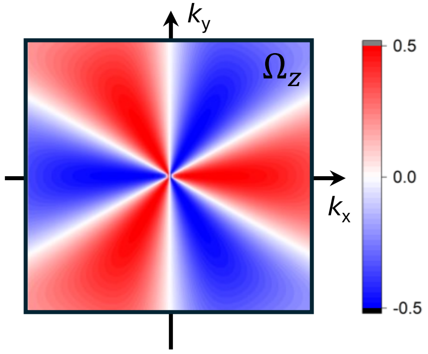
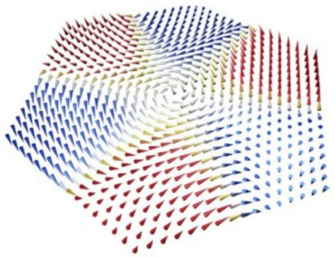




Spin-Momentum Locking



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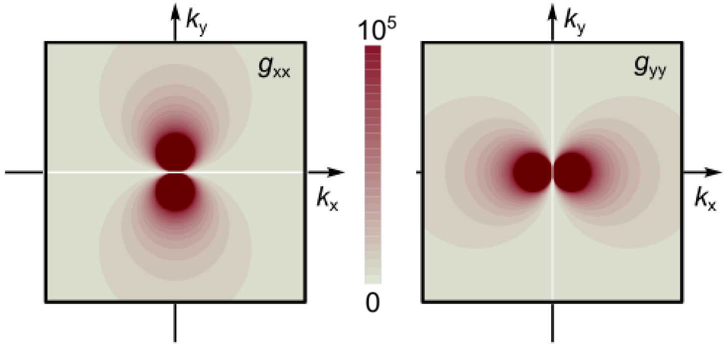
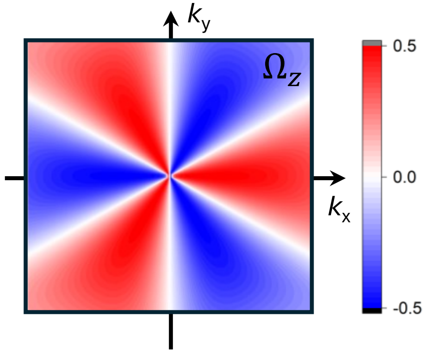
Quantum metric

$$g_{ab} = \frac{1}{4}\partial_{k^a}\hat{\mathbf{h}}\cdot\partial_{k^b}\hat{\mathbf{h}}$$

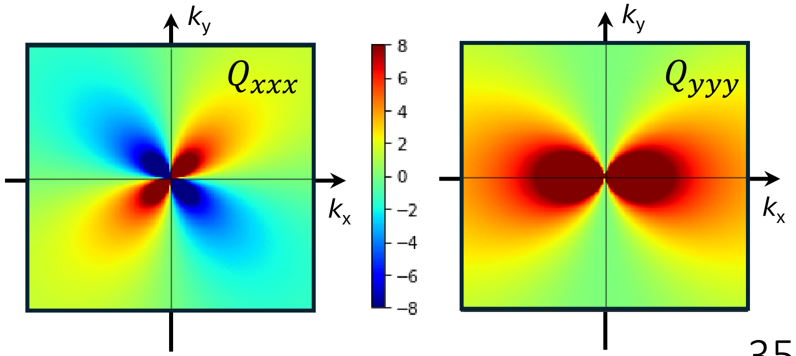


Berry curvature

$$\Omega_{ab} = \frac{\hat{\mathbf{h}}}{2}\cdot\left(\partial_{k^a}\hat{\mathbf{h}}\times\partial_{k^b}\hat{\mathbf{h}}\right)$$



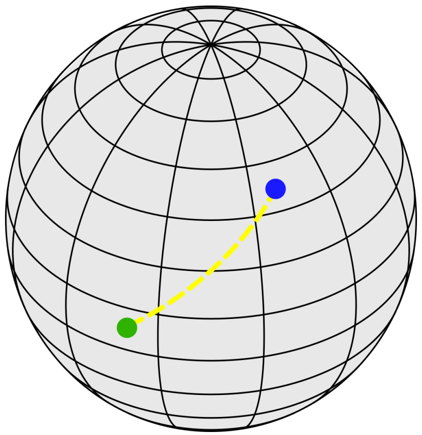
Geometric connection



The Quantum Geometry

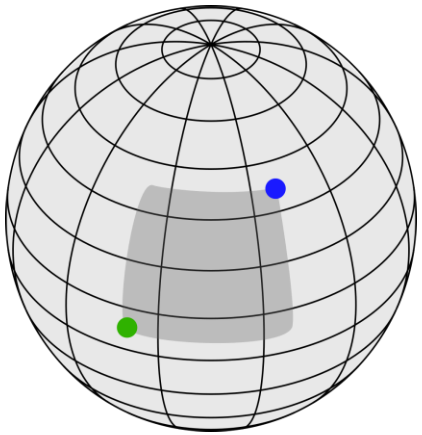
Quantum metric

g_{ab}



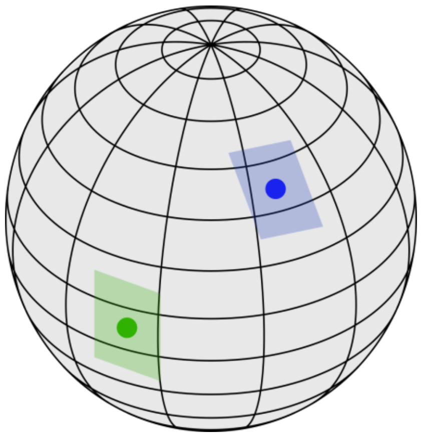
Berry curvature

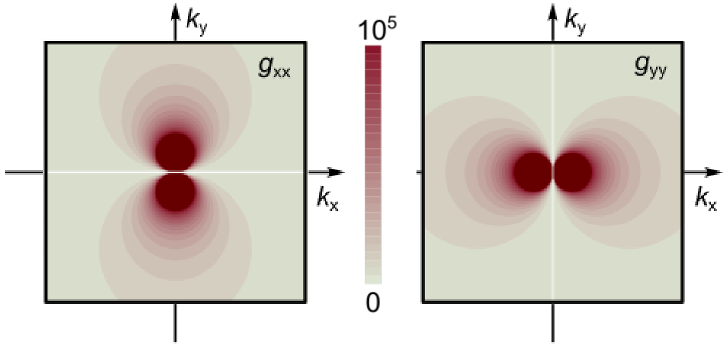
Ω_{ab}

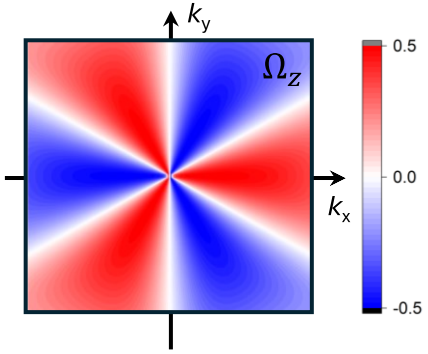


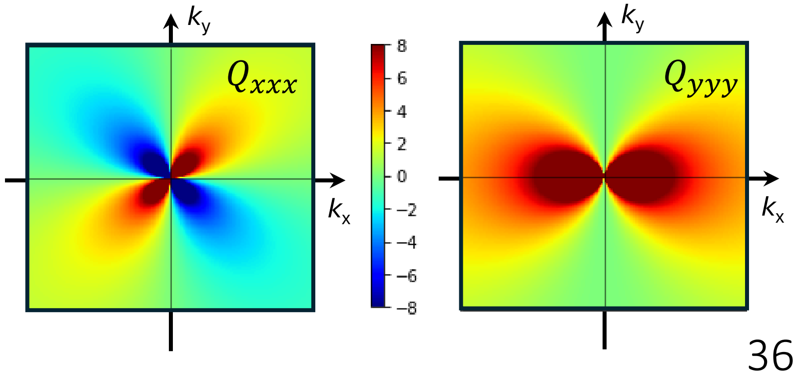
Geometric connection

Q_{abc}









OUTLINE

- ❑ Quantum geometry
- ❑ Spin-momentum locking
- ❑ Linear and nonlinear magnetotransport

NONLINEAR ELECTRONIC TRANSPORT

$$j_a = \sigma_{ab} E_b + \sigma_{abc} E_b E_c + \sigma_{abcd} E_b E_c E_d \ldots$$

1st

$$\sigma^{a;b} = \frac{e^2}{\hbar} \int_k \sum_n f_n \Omega_n^{ab} - \frac{e^2 \tau}{\hbar^2} \int_k \sum_n \frac{\partial f_n}{\partial k_a} \frac{\partial \varepsilon_n}{\partial k_b}$$

2nd

$$\begin{aligned} \sigma^{ab;c} = & -\frac{e^3 \tau^2}{\hbar^3} \sum_n \int_{\boldsymbol{k}} f_n \partial_{k^a} \partial_{k^b} \partial_{k^c} \varepsilon_n \\ & + \frac{e^3 \tau}{\hbar^2} \sum_n \int_{\boldsymbol{k}} f_n \frac{1}{2} \left(\partial_{k^a} \Omega_n^{bc} + \partial_{k^b} \Omega_n^{ac} \right) \\ & - \frac{e^3}{\hbar} \sum_n \int_{\boldsymbol{k}} f_n \left(2 \partial_{k^c} G_n^{ab} - \frac{1}{2} \left(\partial_{k^a} G_n^{bc} + \partial_{k^b} G_n^{ac} \right) \right) \end{aligned}$$

3rd

$$\begin{aligned} \sigma^{abc;d} = & \frac{e^4 \tau^3}{\hbar^4} \sum_n \int_{\boldsymbol{k}} f_n \partial_{k^a} \partial_{k^b} \partial_{k^c} \partial_{k^d} \varepsilon_n \\ & - \frac{e^4 \tau^2}{\hbar^3} \sum_n \int_{\boldsymbol{k}} f_n \frac{1}{3} \left(\partial_{k^a} \partial_{k^b} \Omega_n^{cd} + \partial_{k^b} \partial_{k^c} \Omega_n^{ad} + \partial_{k^a} \partial_{k^c} \Omega_n^{ba} \right) \\ & + \frac{e^4 \tau}{\hbar^2} \sum_n \int_{\boldsymbol{k}} f_n \frac{1}{3} \Big(2 \left(\partial_{k^a} \partial_{k^d} G_n^{bc} + \partial_{k^b} \partial_{k^d} G_n^{ac} + \partial_{k^c} \partial_{k^d} G_n^{ab} \right) \\ & \qquad - \left(\partial_{k^a} \partial_{k^c} G_n^{bd} + \partial_{k^b} \partial_{k^c} G_n^{ad} + \partial_{k^a} \partial_{k^b} G_n^{cd} \right) \Big) \\ & \frac{e^4}{\hbar^3} \sum_{m,p,\boldsymbol{k}} \frac{1}{\omega_{mp}^2} \left[\Gamma_{mp}^{abc} \partial_d f_m + \tilde{\Gamma}_{mp}^{\overline{adb}} \partial_c f_m + \tilde{\Gamma}_{mp}^{\overline{acd}} \partial_b f_m - \frac{1}{3} \tilde{\Gamma}_{mp}^{\overline{bcd}} \partial_a f_m \right] \end{aligned}$$

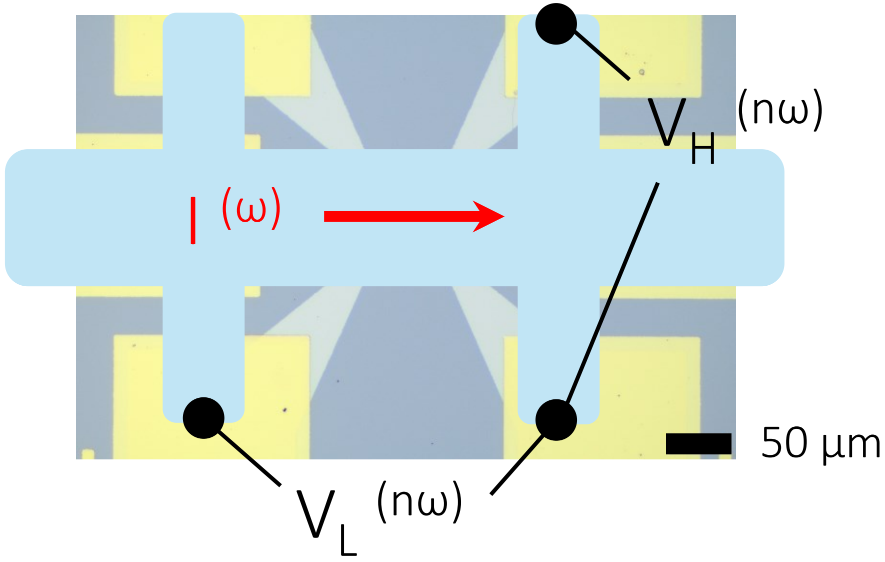
Mandal, Phys. Rev. B 110, 195131, 2024
Fang, Phys. Rev. Lett. 133, 106701, 2024

Nonlinear Electronic Transport

Nonlinear DC Response									
(I) Frequency	$\hbar\omega > E_g$				$\hbar\omega < E_g$		$\omega\tau \ll 1$		
	Nonlinear Optics				Subgap		Nonlinear Transport		
(II) Lifetime	$\mathcal{O}(\tau^1)$		$\mathcal{O}(\tau^0)$		$\frac{\tau_{\text{inter}}}{\tau_{\text{intra}}} \neq 2$	$\frac{\tau_{\text{inter}}}{\tau_{\text{intra}}} = 2$	$\mathcal{O}(\tau^2)$	$\mathcal{O}(\tau^1)$	$\mathcal{O}(\tau^0)$
	Injection current		Shift current		Weak Correlation/ Localization	Strong Correlation/ Localization	Nonlinear Drude	Berry curvature dipole	Quantum metric dipole
(III) Symmetry	$\sigma^{(a,b);c}$	$\sigma^{[a,b];c}$	$\sigma^{(a,b);c}$	$\sigma^{[a,b];c}$	$\mathcal{T}$-breaking		$\sigma^{(a,b;c)}$	$\sigma^{(a,b);c}$	$\sigma^{(a,b);c}$
	$\mathcal{PT}$	$\mathcal{T}$	$\mathcal{T}$	$\mathcal{PT}$			$\mathcal{PT}$	$\mathcal{T}$	$\mathcal{PT}$
(IV) Phenomenon	Linear injection [22, 23]	Circular injection [24–31]	Linear shift [32–51]	Circular shift [23, 52, 53]	Subgap response [54–57]		Nonreciprocal magneto-resistance (NMR) [58–61]	Nonlinear Anomalous Hall (NLAH) [58, 62–67]	Intrinsic NMR & NLAH [68–73]
(V) Quantum Geometry	Quantum metric	Berry curvature	Symplectic Christoffel symbol	Christoffel symbol of 1st kind	-		Translation	Rotation	Distortion

Jiang, arXiv2503.04943v1

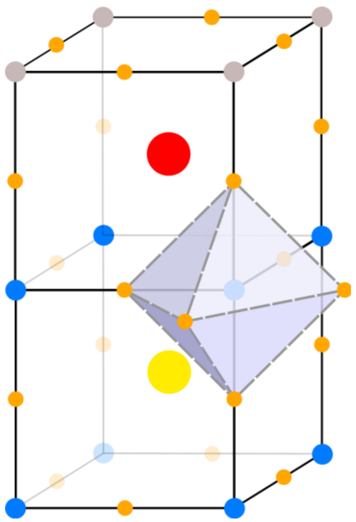
NONLINEAR ELECTRONIC TRANSPORT

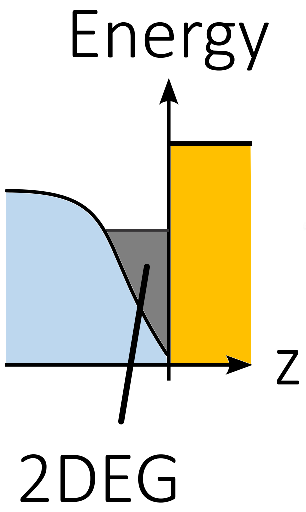


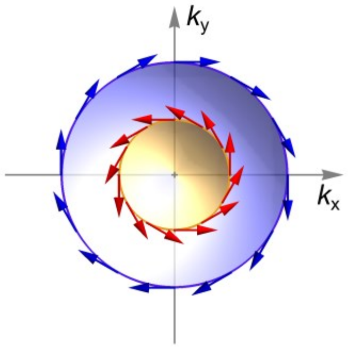
- Lock-in amplifiers
- Cryogenic temperatures
- Magnetic fields
- Gate voltages

Nonlinear Electronic Transport

$LaAlO_3$ / $SrTiO_3$

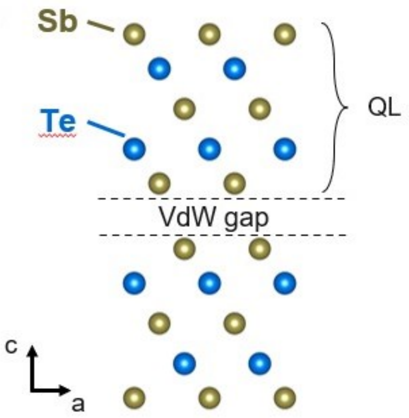


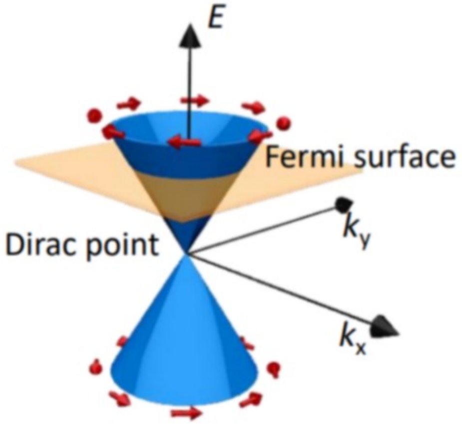




Ohtomo, Nature 427, 423, 2004
Reyren, Science 317, 1196, 2007
Caviglia, Nature 456, 624, 2008

Sb_2Te_3





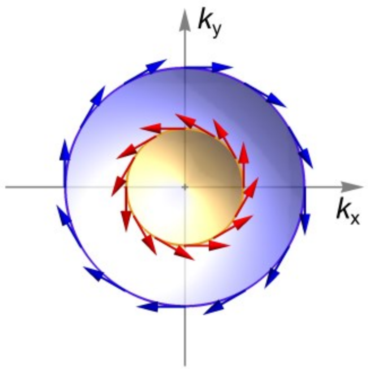
König, Science 318, 5851, 2007
Hasan, Rev. Mod. Phys. 82, 3045, 2010

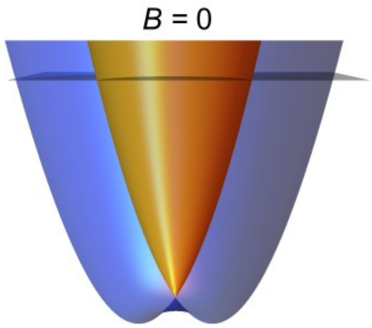
Berry Curvature – 1ω

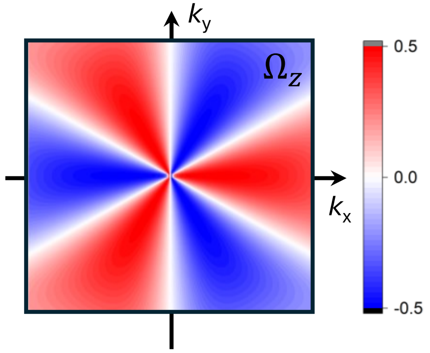
LaAlO$_3$ / SrTiO$_3$

1st

$$\sigma^{a;b} = \frac{e^2}{\hbar}\int_k \sum_n f_n \Omega_n^{ab} - \frac{e^2\tau}{\hbar^2}\int_k \sum_n \frac{\partial f_n}{\partial k_a}\frac{\partial \varepsilon_n}{\partial k_b}$$





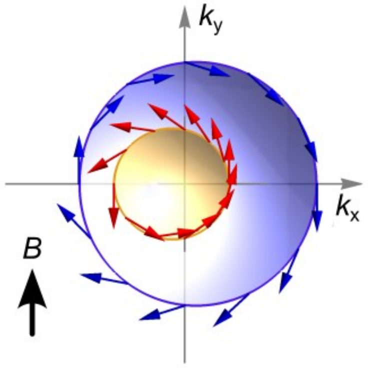


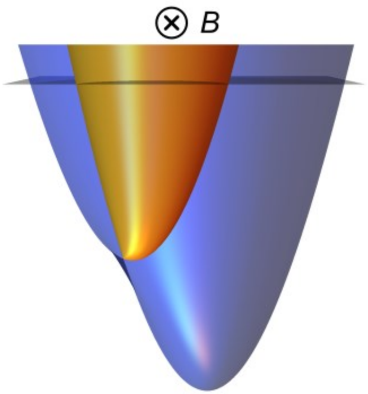
Berry Curvature – 1ω

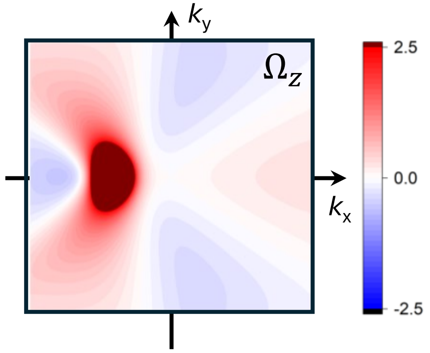
LaAlO$_3$ / SrTiO$_3$

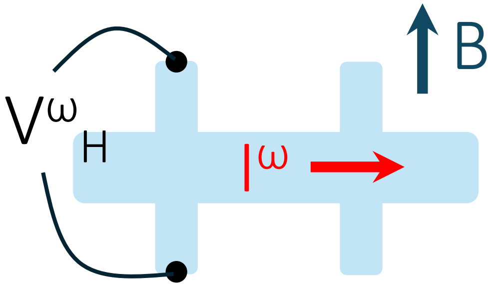
1st

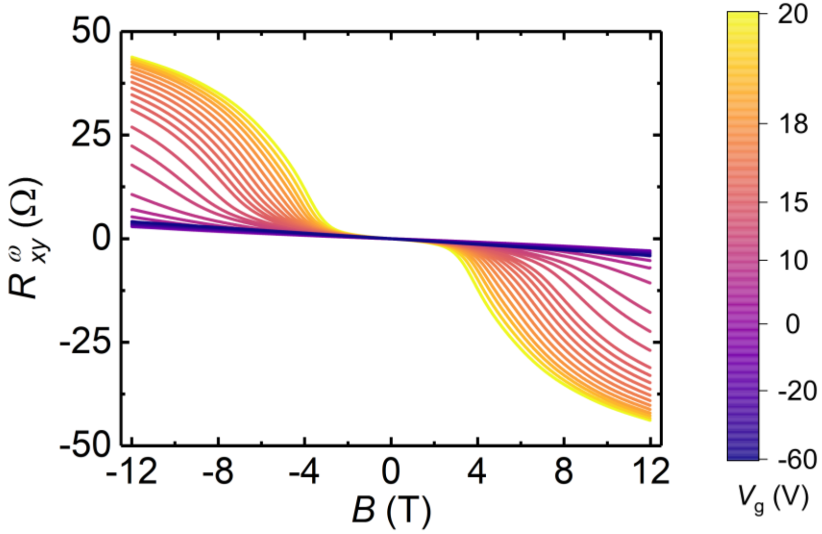
$$\sigma^{a;b} = \frac{e^2}{\hbar}\int_k \sum_n f_n \Omega_n^{ab} - \frac{e^2\tau}{\hbar^2}\int_k \sum_n \frac{\partial f_n}{\partial k_a}\frac{\partial \varepsilon_n}{\partial k_b}$$









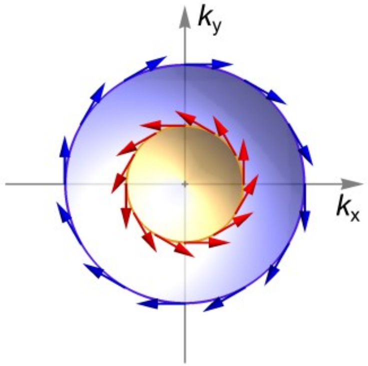


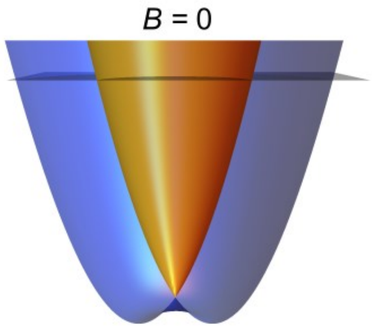
Quantum Metric – 2ω

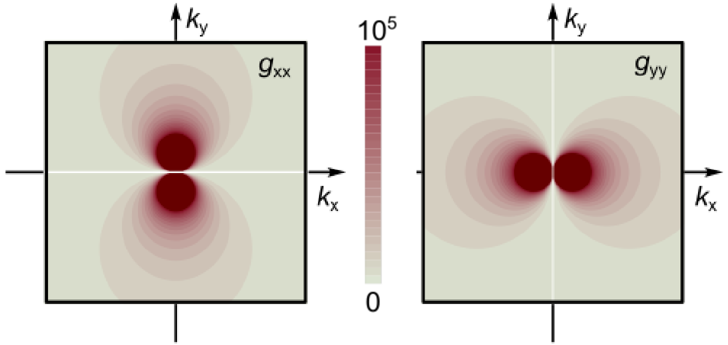
LaAlO$_3$ / SrTiO$_3$

2nd

$$\sigma^{ab;c} = -\frac{e^3\tau^2}{\hbar^3}\sum_n \int_{\mathbf{k}} f_n \partial_{k^a}\partial_{k^b}\partial_{k^c}\varepsilon_n + \frac{e^3\tau}{\hbar^2}\sum_n \int_{\mathbf{k}} f_n \frac{1}{2}\left(\partial_{k^a}\Omega_n^{bc} + \partial_{k^b}\Omega_n^{ac}\right) - \frac{e^3}{\hbar}\sum_n \int_{\mathbf{k}} f_n \left(2\partial_{k^c}G_n^{ab} - \frac{1}{2}\left(\partial_{k^a}G_n^{bc} + \partial_{k^b}G_n^{ac}\right)\right)$$





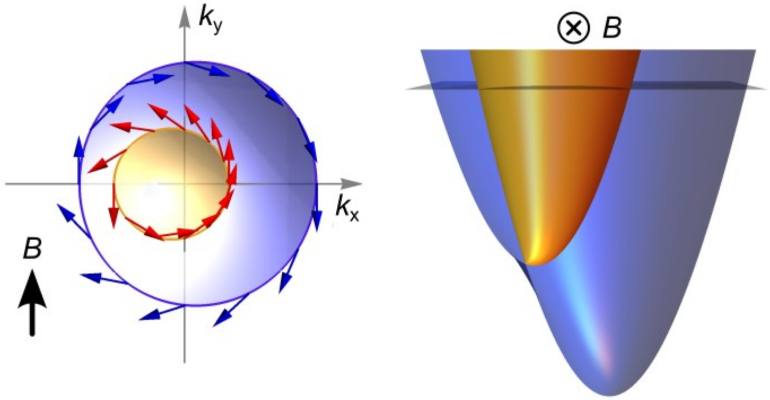


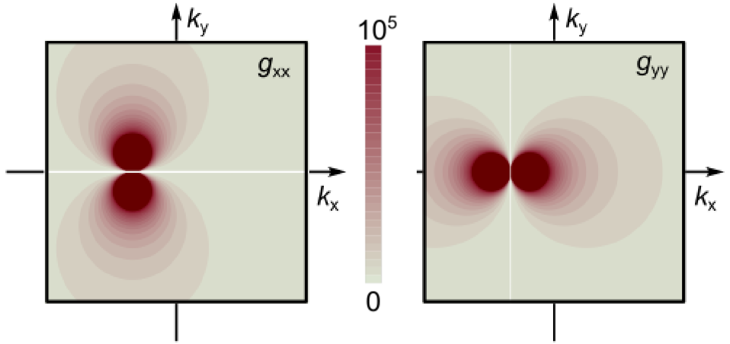
Quantum Metric – 2ω

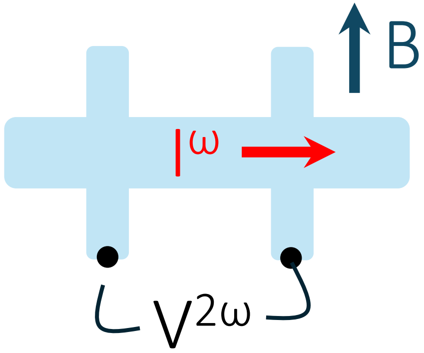
$LaAlO_3$ / $SrTiO_3$

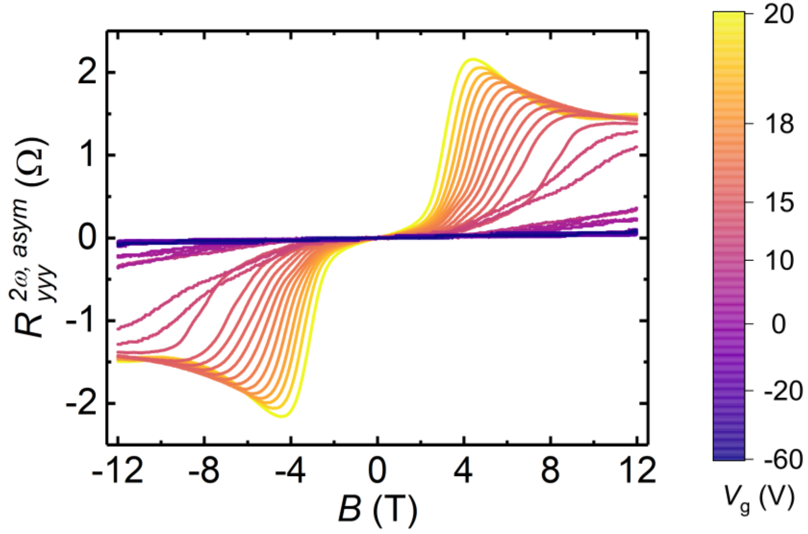
2nd

$$\sigma^{ab;c} = -\frac{e^3\tau^2}{\hbar^3}\sum_n \int_{\boldsymbol{k}} f_n \partial_{k^a}\partial_{k^b}\partial_{k^c}\varepsilon_n + \frac{e^3\tau}{\hbar^2}\sum_n \int_{\boldsymbol{k}} f_n \frac{1}{2}\left(\partial_{k^a}\Omega_n^{bc} + \partial_{k^b}\Omega_n^{ac}\right) - \frac{e^3}{\hbar}\sum_n \int_{\boldsymbol{k}} f_n \left(2\partial_{k^c}G_n^{ab} - \frac{1}{2}\left(\partial_{k^a}G_n^{bc} + \partial_{k^b}G_n^{ac}\right)\right)$$







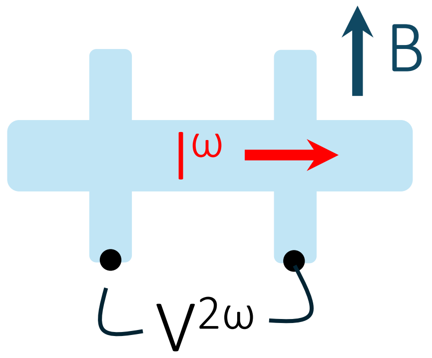


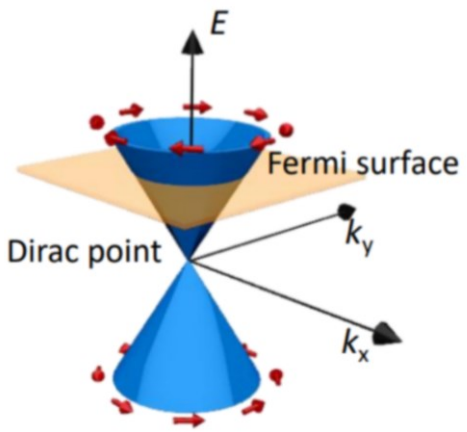
Quantum Metric – 2ω

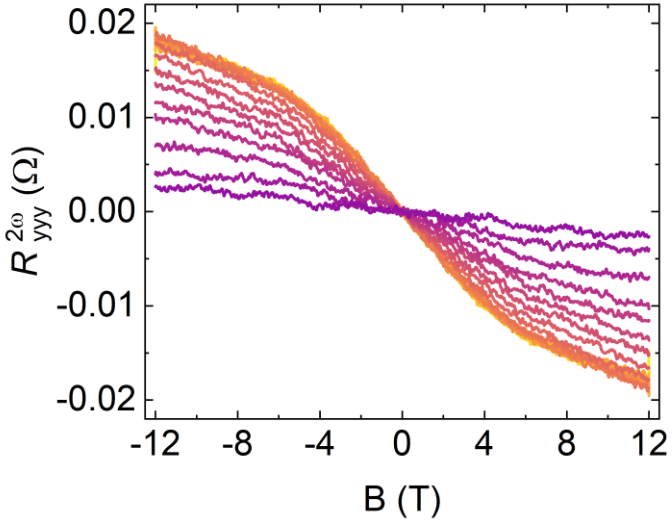
Sb_2Te_3

2nd

$$\sigma^{ab;c} = -\frac{e^3\tau^2}{\hbar^3}\sum_n \int_{\boldsymbol{k}} f_n \partial_{k^a}\partial_{k^b}\partial_{k^c}\varepsilon_n + \frac{e^3\tau}{\hbar^2}\sum_n \int_{\boldsymbol{k}} f_n \frac{1}{2}\left(\partial_{k^a}\Omega_n^{bc} + \partial_{k^b}\Omega_n^{ac}\right) - \frac{e^3}{\hbar}\sum_n \int_{\boldsymbol{k}} f_n \left(2\partial_{k^c}G_n^{ab} - \frac{1}{2}\left(\partial_{k^a}G_n^{bc} + \partial_{k^b}G_n^{ac}\right)\right)$$

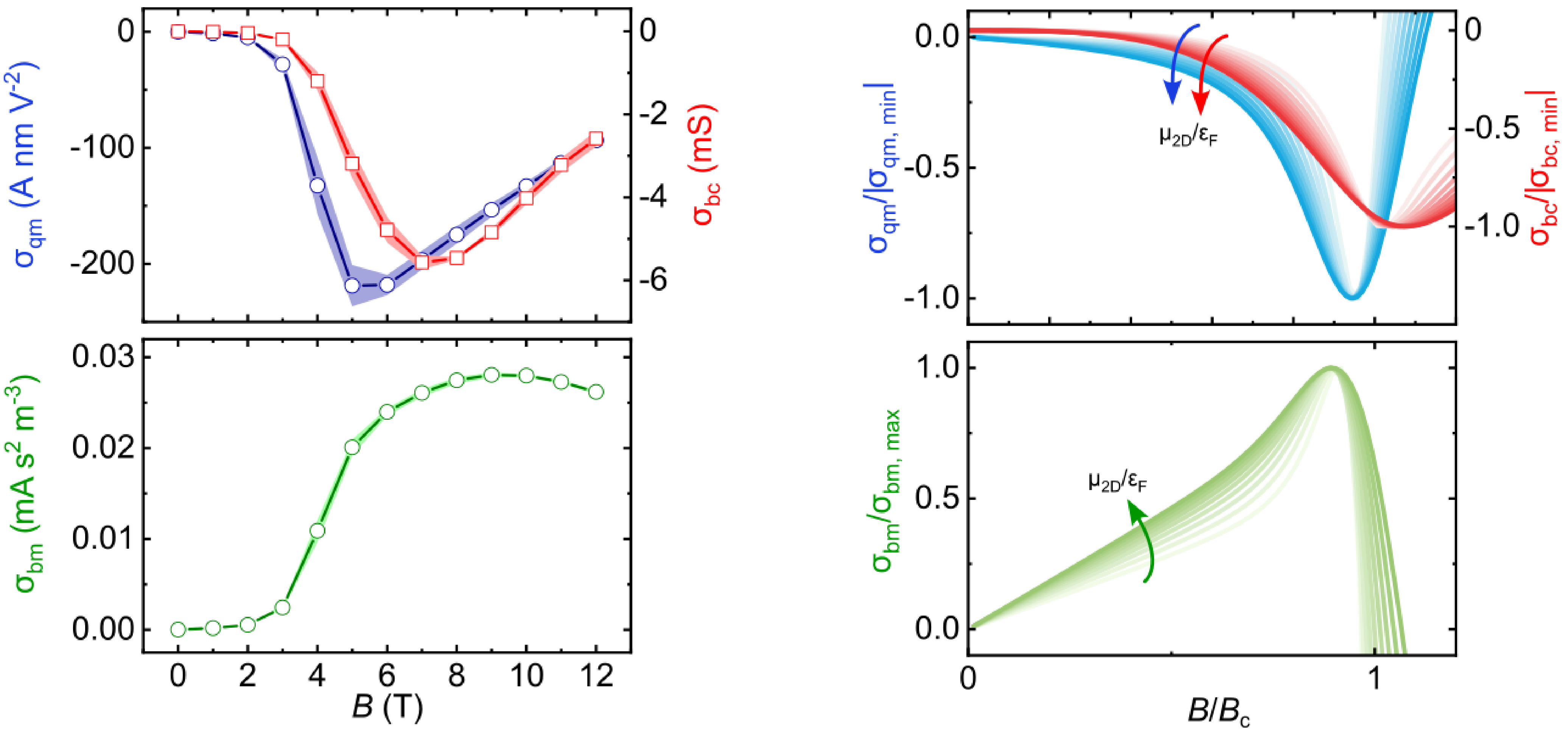






Quantum Geometric Tensor

$$G_{xy} = g_{xy} - \frac{i}{2}\Omega_{xy}$$

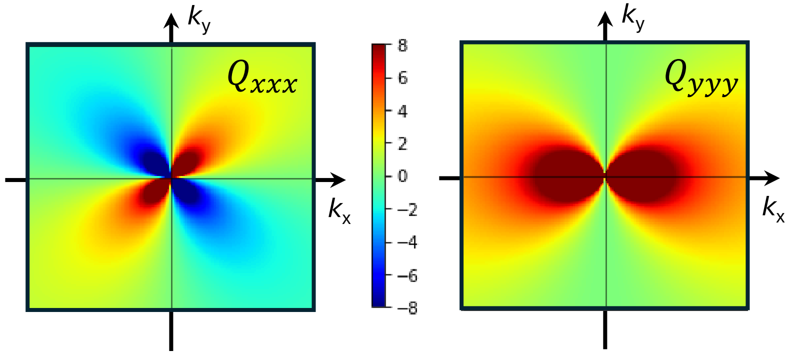


Geometric Connection – 3ω

LaAlO$_3$ / SrTiO$_3$

3rd

$$
\begin{aligned}
\sigma^{abc;d} =& \frac{e^4\tau^3}{\hbar^4}\sum_n \int_{\boldsymbol{k}} f_n \partial_{k^a}\partial_{k^b}\partial_{k^c}\partial_{k^d}\varepsilon_n \\
&- \frac{e^4\tau^2}{\hbar^3}\sum_n \int_{\boldsymbol{k}} f_n \frac{1}{3}\left(\partial_{k^a}\partial_{k^b}\Omega_n^{cd} + \partial_{k^b}\partial_{k^c}\Omega_n^{ad} + \partial_{k^a}\partial_{k^c}\Omega_n^{ba}\right) \\
&+ \frac{e^4\tau}{\hbar^2}\sum_n \int_{\boldsymbol{k}} f_n \frac{1}{3}\Big(2(\partial_{k^a}\partial_{k^d}G_n^{bc} + \partial_{k^b}\partial_{k^d}G_n^{ac} + \partial_{k^c}\partial_{k^d}G_n^{ab}) \\
&\qquad - (\partial_{k^a}\partial_{k^c}G_n^{bd} + \partial_{k^b}\partial_{k^c}G_n^{ad} + \partial_{k^a}\partial_{k^b}G_n^{cd})\Big) \\
&\quad \frac{e^4}{\hbar^3}\sum_{m,p,\boldsymbol{k}} \frac{1}{\omega_{mp}^2}\left[\tilde{\Gamma}_{mp}^{\overline{abc}}\partial_d f_m + \tilde{\Gamma}_{mp}^{\overline{adb}}\partial_c f_m + \tilde{\Gamma}_{mp}^{\overline{acd}}\partial_b f_m - \frac{1}{3}\tilde{\Gamma}_{mp}^{\overline{bcd}}\partial_a f_m\right]
\end{aligned}
$$

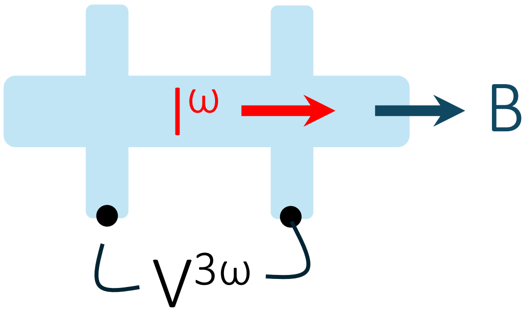


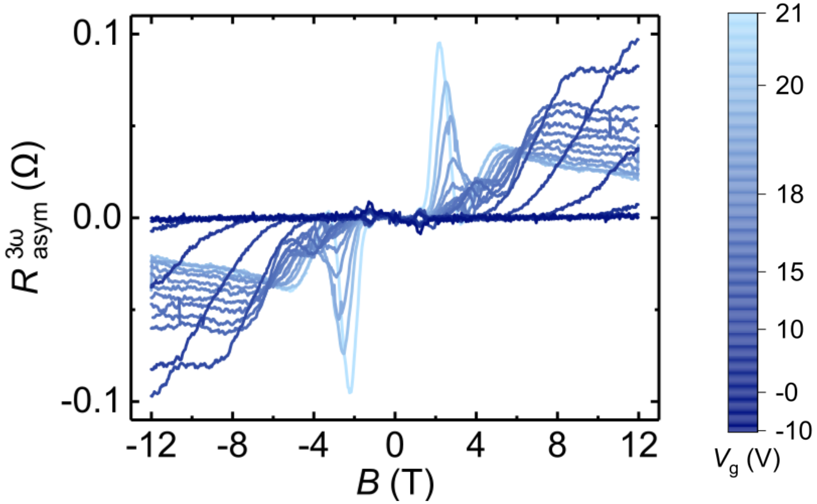
GEOMETRIC CONNECTION – 3ω

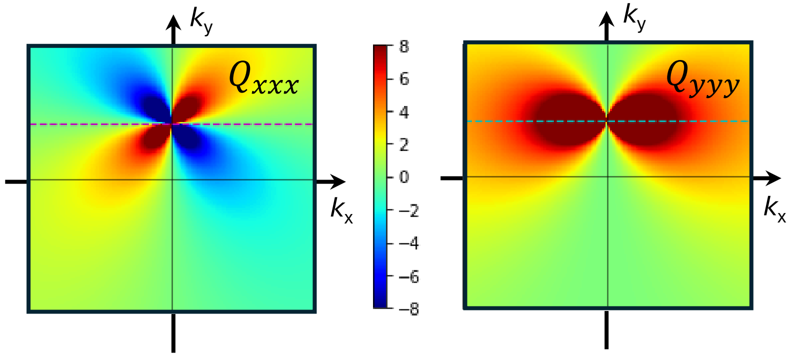
LaAlO$_3$ / SrTiO$_3$

3rd

$$\sigma^{abc;d} = \frac{e^4\tau^3}{\hbar^4}\sum_n \int_{\boldsymbol{k}} f_n \partial_{k^a}\partial_{k^b}\partial_{k^c}\partial_{k^d}\varepsilon_n$$
$$- \frac{e^4\tau^2}{\hbar^3}\sum_n \int_{\boldsymbol{k}} f_n \frac{1}{3}\left(\partial_{k^a}\partial_{k^b}\Omega_n^{cd} + \partial_{k^b}\partial_{k^c}\Omega_n^{ad} + \partial_{k^a}\partial_{k^c}\Omega_n^{bd}\right)$$
$$+ \frac{e^4\tau}{\hbar^2}\sum_n \int_{\boldsymbol{k}} f_n \frac{1}{3}\Big(2(\partial_{k^a}\partial_{k^d}G_n^{bc} + \partial_{k^b}\partial_{k^d}G_n^{ac} + \partial_{k^c}\partial_{k^d}G_n^{ab})$$
$$- (\partial_{k^a}\partial_{k^c}G_n^{bd} + \partial_{k^b}\partial_{k^c}G_n^{ad} + \partial_{k^a}\partial_{k^b}G_n^{cd})\Big)$$
$$\frac{e^4}{\hbar^3}\sum_{m,p,\boldsymbol{k}} \frac{1}{\omega_{mp}^2}\left[\tilde{\Gamma}_{mp}^{abc}\partial_d f_m + \tilde{\Gamma}_{mp}^{\overline{adb}}\partial_c f_m + \tilde{\Gamma}_{mp}^{\overline{acd}}\partial_b f_m - \frac{1}{3}\tilde{\Gamma}_{mp}^{\overline{bcd}}\partial_a f_m\right]$$

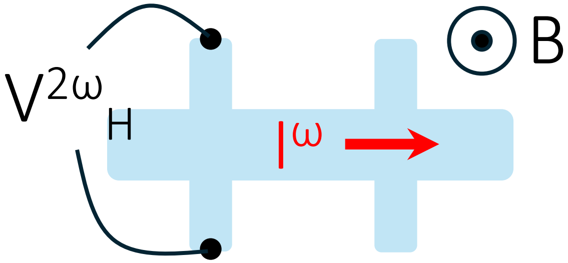


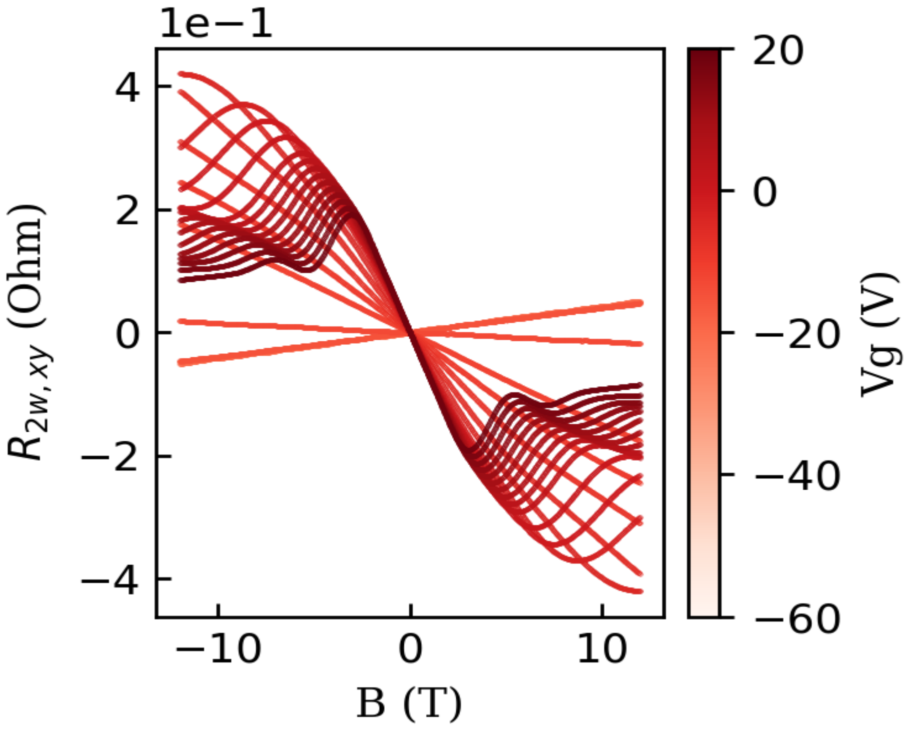




BEYOND QUANTUM GEOMETRY – NONLINEAR ORBITRONICS

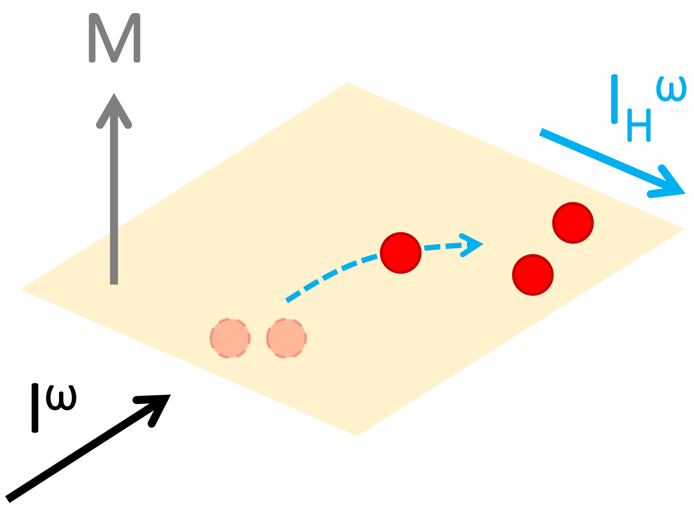
LaAlO$_3$ / SrTiO$_3$



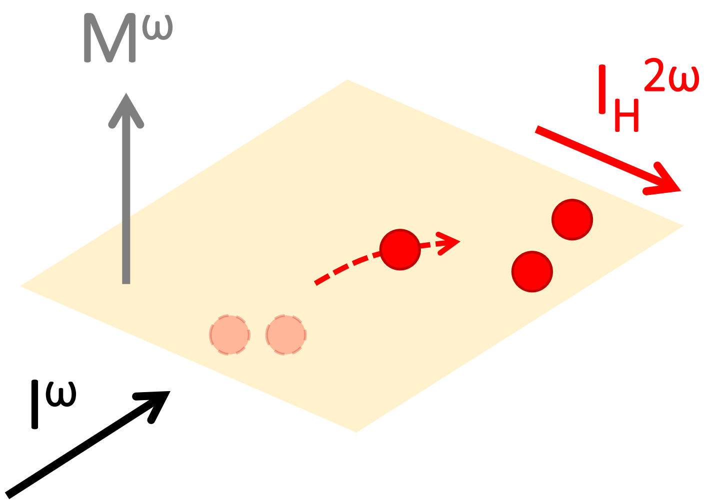


BEYOND QUANTUM GEOMETRY – NONLINEAR ORBITRONICS

Anomalous Hall



Nonlinear anomalous Hall

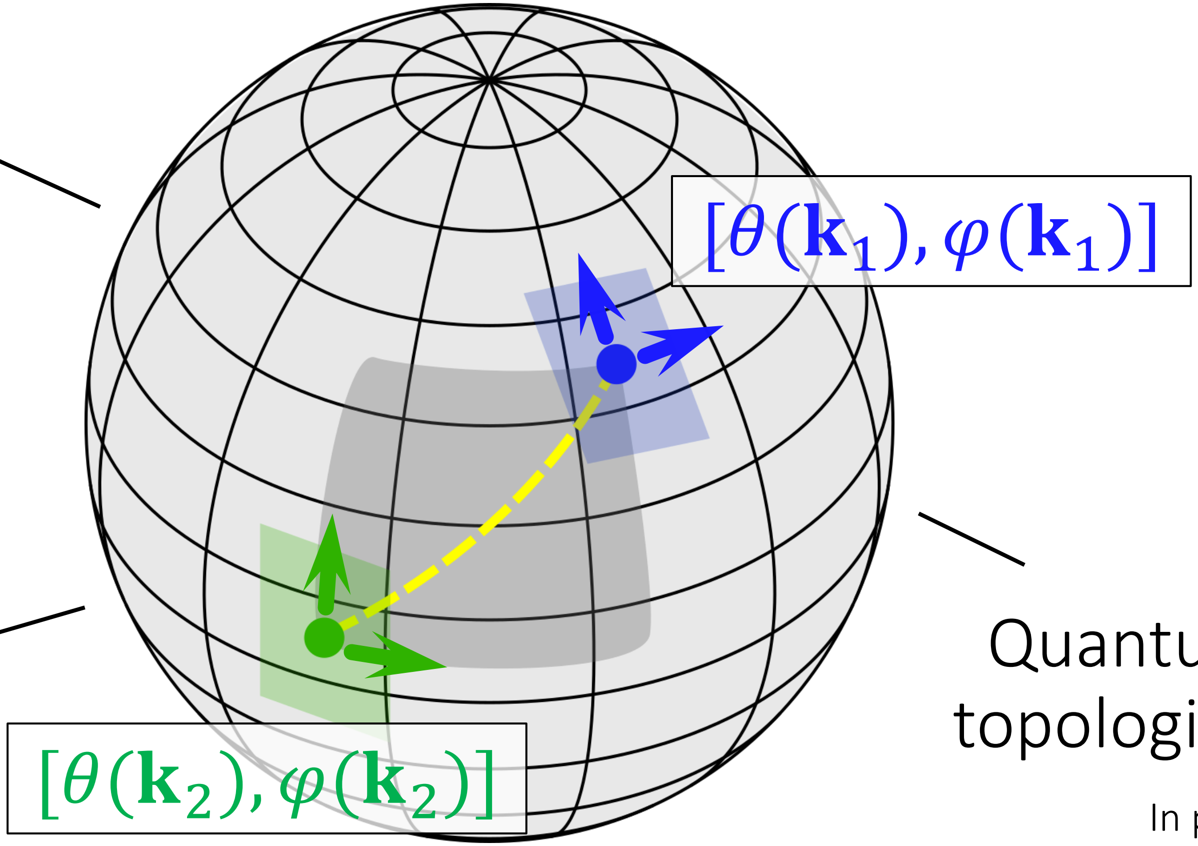


Nagaosa, Rev. Mod. Phys. 82, 1539, 2010

SUMMARY & OUTLOOK

Berry curvature & quantum metric in Rashba 2DEGs

https://arxiv.org/abs/2407.06659
accepted in Science



Geometric connection

In preparation

Quantum metric in topological insulators

In preparation

ACKNOWLEDGMENTS

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S. Gariglio

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D. Mandal

E. Longo

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