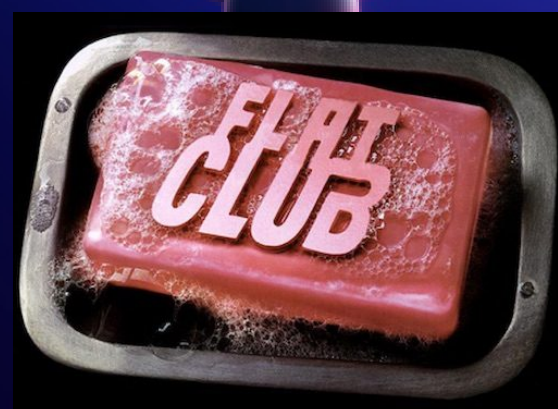
DQMP Department of Quantum Matter Physics

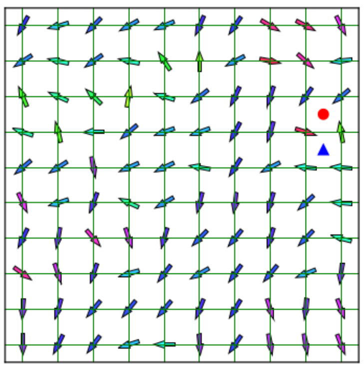
The struggle of 2d superconductors

Giulia Venditti



Previously on Flat Club…

Summary: 2D superconductors



Described by **XY model**

No true long-range order

Resistivity governed by **vortices**
(BKT transition, SIT transition)

Open Questions:

- What happens to the **Cooper pairs** above a BKT transition?
- Role of vortices in **unconventional** order parameters (d-wave, topological)?
- Nature of the **Quantum metallic state**?
- What is the **glue**?
- Role of vortex **pinning**?

Louk Rademaker

2D superconductivity

22 October 2021

Today on Flat Club...

Giulia Venditti
2D superconductivity
15 March 2024

- Quick review of **BKT**
- BKT and <u>real</u> **superconductors**
- BKT vs inhomogeneities

Open Questions:

- What happens to the **Cooper pairs** above a BKT transition?
- Role of vortices in **unconventional** order parameters (d-wave, topological)?
- Nature of the **Quantum metallic state**?
- What is the **glue**?
- Role of vortex **pinning**?

New questions

Superconductivity

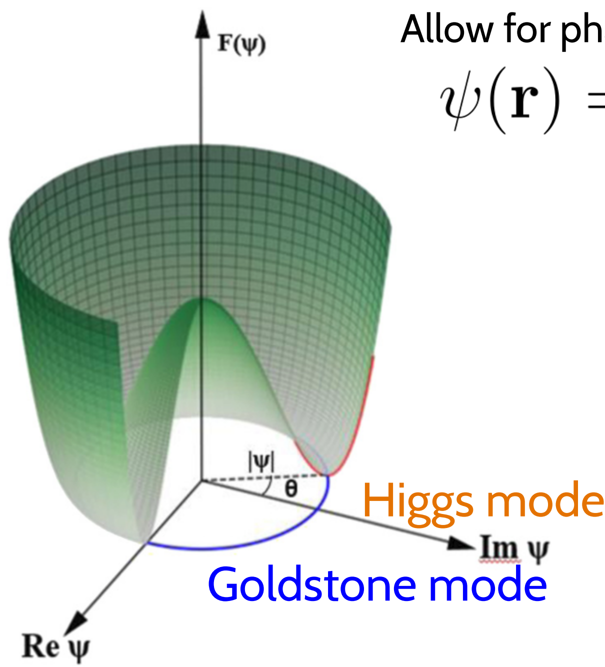
$$\psi = |\psi| e^{i\theta}$$

- Formation of Cooper pairs $\psi = \langle c^\dagger_{k\sigma} c^\dagger_{k'\sigma'} \rangle$
- Global phase rigidity of the manybody system

Superfluid stiffness:
the energetic cost to twist the phase

$$J_s \propto n_s$$

- Zero resistivity
- Meissner effect

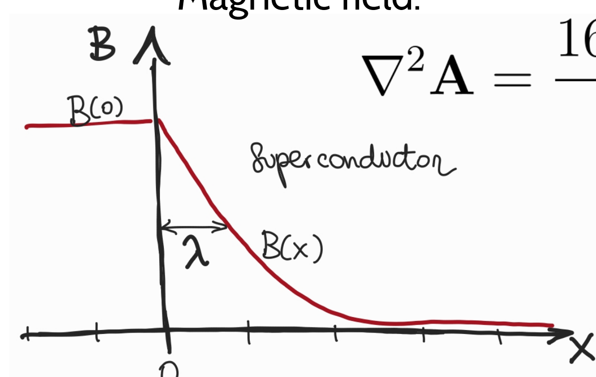


Allow for phase fluctuations:

$$\psi(\mathbf{r}) = \psi_0 e^{i\theta(\mathbf{r})}$$

$$f = f_0 + \alpha(T - T_c)\psi_0^2 + \beta\psi_0^4 + \frac{\hbar^2\psi_0^2}{2m^*}|\nabla\theta(\mathbf{r})|^2$$

Magnetic field:



$$\nabla^2 \mathbf{A} = \frac{16\pi e^2 \psi_0^2}{m^* c^2}\mathbf{A} \equiv \frac{1}{\lambda^2}\mathbf{A}$$

London penetration depth

BKT transition

$$\psi = |\psi| e^{i\theta}$$

- Formation of Cooper pairs $\psi = \langle c^{\dagger}_{k\sigma} c^{\dagger}_{k'\sigma'} \rangle$
- Global phase rigidity of the manybody system

Superfluid stiffness:
the energetic cost to twist the phase

$$J_s \propto n_s$$

- Zero resistivity
- Meissner effect

No ext fields nor currents

(discrete) XY model

$$H \propto J_s \int |\nabla\theta(\mathbf{r})|^2 d\mathbf{r} \quad \Longleftrightarrow \quad H = -J \sum_{<i,j>} \mathbf{s}_i \cdot \mathbf{s}_j$$

Mermin-Wagner theorem: no long range order in d<3 $\langle \psi \rangle = 0$

Berezinskii–Kosterlitz–Thouless transition!

Nobel prize 2016

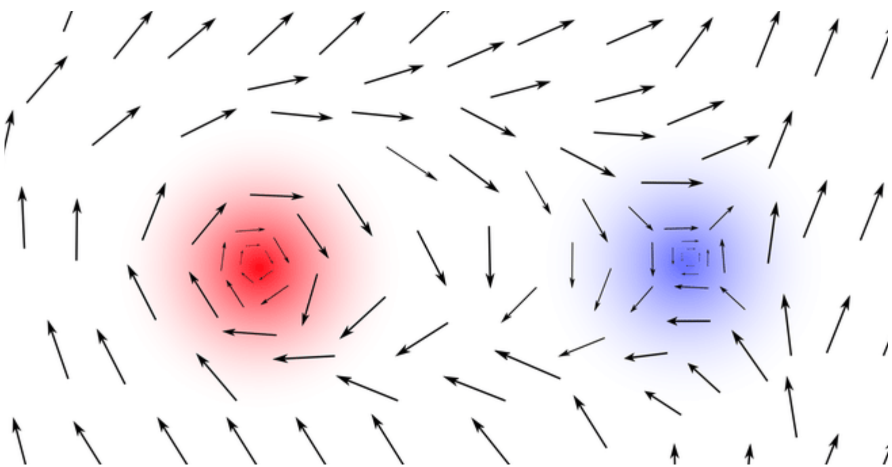
$$J_S \propto \langle \psi^2 \rangle \propto \lambda^{-2}$$

Pratap Raychaudhuri and Surajit Dutta, J. Phys.: Condens. Matter 34 (2022) 083001

BKT transition

Spontaneous Symmetry Breaking: long range order $\langle\psi\rangle \neq 0$

BKT topological phase transition: quasi-long-range order $\langle\psi\rangle = 0$, $\langle\psi^2\rangle \neq 0$



- continuous global symmetry *U(1)*

$$\theta_i \to \theta_i + \chi \quad \forall i$$

- a discrete local symmetry $\mathbb{Z}^m$

$$\theta_i \to \theta_i + 2\pi m \quad \forall m \in \mathbb{Z}$$

Winding number m=0, ±1, ±2, ±3, ...

$$\oint \nabla\theta \cdot d\ell = \pm 2\pi m$$

Topological excitations: vortex-antivortex pairs

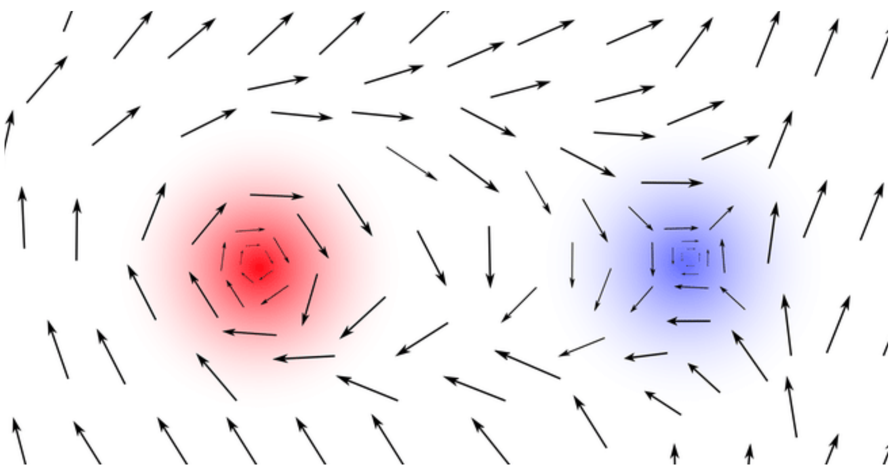
BKT transition

Spontaneous Symmetry Breaking: long range order $\langle\psi\rangle \neq 0$

$$T \neq T_c \quad C(\mathbf{r}) \sim \mathrm{e}^{-|\mathbf{r}|/\xi} \quad \xi \propto |T - T_c|^{-\nu}$$

BKT topological phase transition: quasi-long-range order $\langle\psi\rangle = 0\,,\ \langle\psi^2\rangle \neq 0$

Topological excitations: vortex-antivortex pairs



$T > T_{\mathrm{BKT}}$ $\quad C^{xy}(\mathbf{r}) \sim \mathrm{e}^{-|\mathbf{r}|/\xi} \quad \xi \sim \dfrac{1}{\ln(2T/\mathcal{J})}$

$T \sim T^{+}_{\mathrm{BKT}}$ $\quad \xi_+ \sim e^{b/\sqrt{T-T_{\mathrm{BKT}}}}$

$T \leq T_{\mathrm{BKT}}$

$$C^{xy}(\mathbf{r}) \sim \left(\frac{a}{|\mathbf{r}|}\right)^{\frac{T}{2\pi\mathcal{J}}}$$

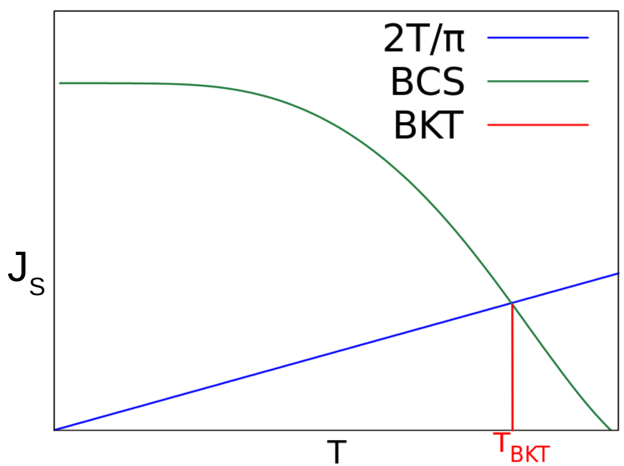
$\xi \to \infty$

always critical!

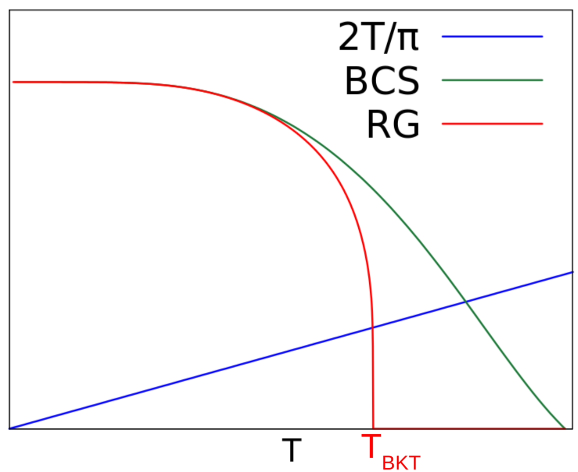
Mermin-Wagner theorem: diverging fluctuations! BUT something is happening

BKT transition

Ideal case



Renormalized



A naive estimate of T_{BKT}:

Single vortex in a lattice (size L, lattice const. a)

$$E_\mathrm{v} \sim J\int_a^L d\mathbf{r}(\nabla\theta(\mathbf{r}))^2 = \pi J\log(\frac{L}{a})$$

$$S_\mathrm{v} = 2k_\mathrm{B}\log(\frac{L}{a})$$

$$F_\mathrm{v} = E_\mathrm{v} - T_\mathrm{BKT}S_\mathrm{v} = 0$$

Jump of the superfluid stiffness at the critical point

$$J_s(T_{BKT}) = \frac{2T_{BKT}}{\pi}$$

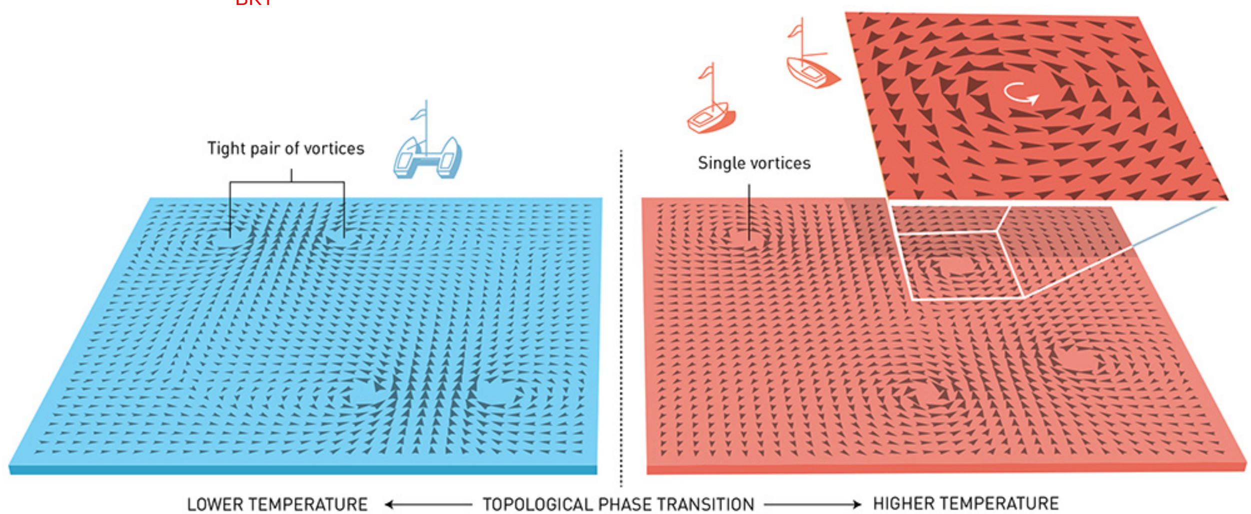


Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

BKT transition

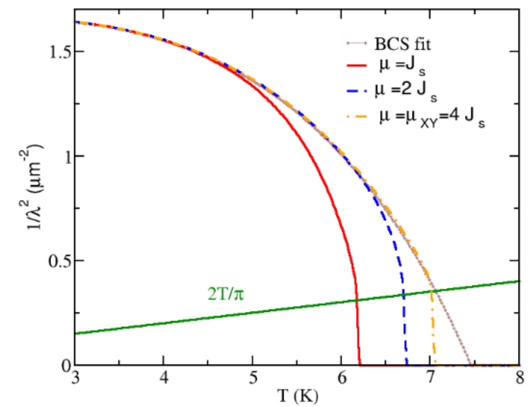
RG equations
(map to Sine-Gordon model)

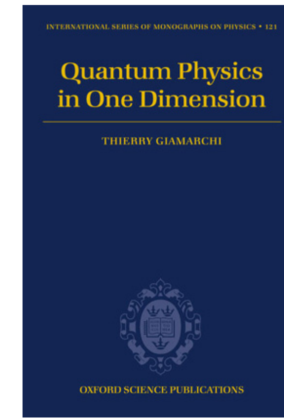
$$\begin{cases} K = \dfrac{\pi J_s}{T} \\ g = 2\pi e^{-\beta\mu_{\mathrm{v}}} \end{cases}$$

Vortex-core energy
$\mu_V = \pi\xi_0^2\epsilon_{\mathrm{cond}}$

$$\Lambda(\ell) = \Lambda_0 e^{-\ell},\ \ell = \ln(\frac{L}{a})$$

$$\begin{cases} \dfrac{dK}{d\ell} = -K^2 g^2 \\ \dfrac{dg}{d\ell} = (2-K)g \end{cases}$$





$$K(\ell \to \infty) = 2 \longrightarrow J_{\mathrm{S}}(T_{\mathrm{BKT}}) = \frac{2T_{\mathrm{BKT}}}{\pi}$$

Details of the model goes into initial values of J_s and μ_v

¡LR? What is the **glue**?
Role of vortices in **unconventional order parameters**?

$$\Psi(r,\theta) = \Psi_0(r)\, e^{i\theta}$$

U(1)

Amplitude fluctuations affects the initial value of the RG flow ($\to J_s$ and T_{BKT})

not the critical point

But don't mess up with vortex-antivortex pairs…

PHYSICAL REVIEW B VOLUME 62, NUMBER 10 1 SEPTEMBER 2000-II

Effective actions and phase fluctuations in *d*-wave superconductors

Arun Paramekanti,[1] Mohit Randeria,[1] T. V. Ramakrishnan,[2] and S. S. Mandal[2,3]

PHYSICAL REVIEW B **102**, 104505 (2020)

Interplay of spin waves and vortices in the two-dimensional XY model at small vortex-core energy

I. Maccari,[1] N. Defenu,[2,3] L. Benfatto,[4,5] C. Castellani,[4,5] and T. Enss[2]

¡GV?

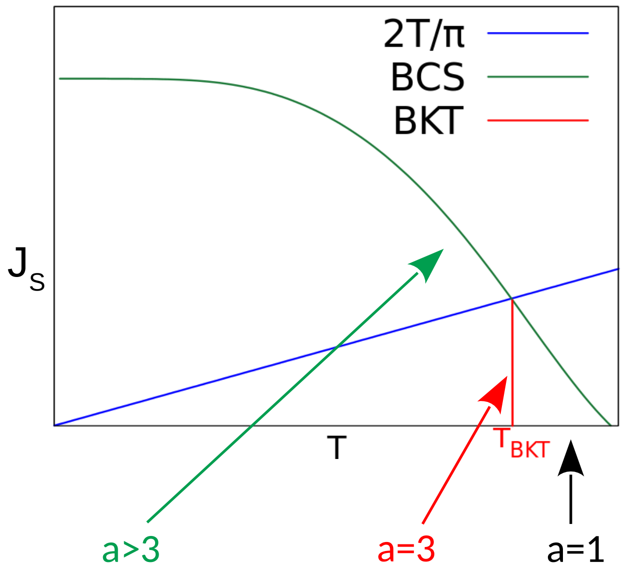
...Time for discussion

"Third Newton's rule:
If you provoke me, I'll beat you up"

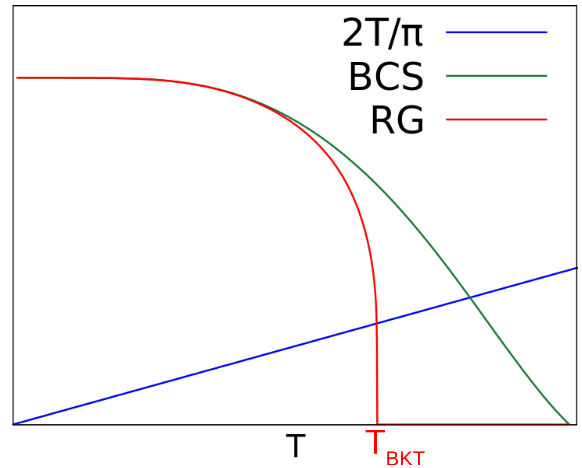
@la_scienza_coatta

BKT transition

Ideal case



Renormalized



Jump of the superfluid stiffness at the critical point

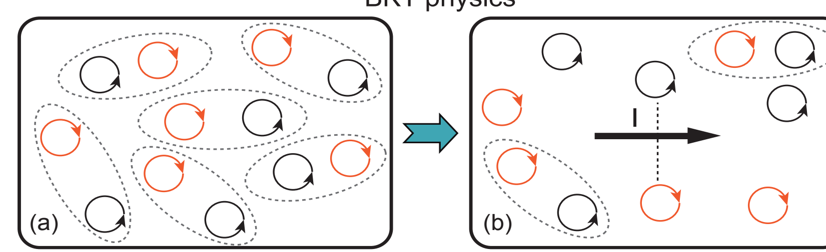
$$J_s(T_{BKT}) = \frac{2T_{BKT}}{\pi}$$

Non-linear *IV* characteristics

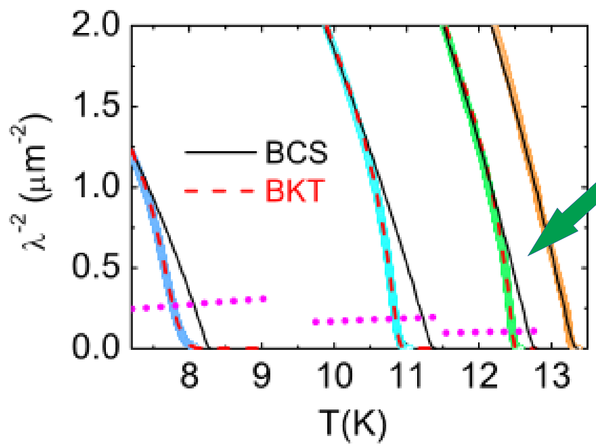
$$V \propto I^{a(T)}$$

$$a(T) = 1 + \frac{\pi J_s}{T} = \begin{cases} 1, \text{ if } T > T_{BKT} \\ 3, \text{ if } T = T_{BKT} \\ > 3, \text{ if } T < T_{BKT} \end{cases}$$

BKT physics



Observation of BKT signatures



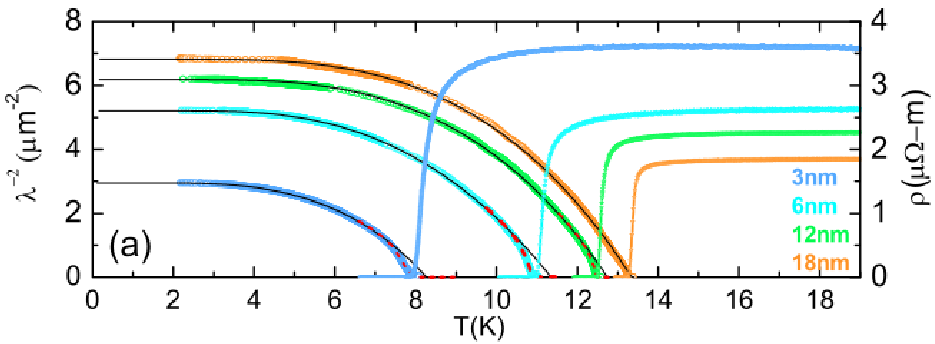
T_{BKT} and T_c almost indistinguishable

$$\frac{T_c - T_{BKT}}{T_{BKT}} \approx \frac{8}{\pi^3}\frac{T_c}{J_0}$$

s-wave

- Reducing d→ increase disorder, suppress J_0
- Smearing of the jump caused by disorder (~nm)

Uncorrelated disorder does not affect the critical behavior!



PRL **107**, 217003 (2011) PHYSICAL REVIEW LETTERS week ending 18 NOVEMBER 2011

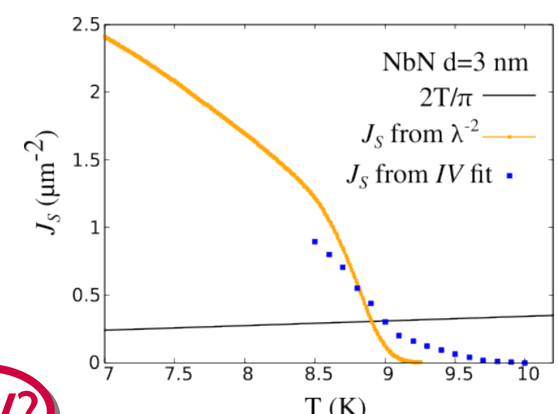
Role of the Vortex-Core Energy on the Berezinskii-Kosterlitz-Thouless Transition in Thin Films of NbN

Mintu Mondal,[1] Sanjeev Kumar,[1] Madhavi Chand,[1] Anand Kamlapure,[1] Garima Saraswat,[1] G. Seibold,[2] L. Benfatto,[3,*] and Pratap Raychaudhuri[1,†]

PHYSICAL REVIEW B **100**, 064506 (2019)

Nonlinear *I-V* characteristics of two-dimensional superconductors: Berezinskii-Kosterlitz-Thouless physics versus inhomogeneity

G. Venditti,[1] J. Biscaras,[2] S. Hurand,[3,4] N. Bergeal,[3,5] J. Lesueur,[3,5] A. Dogra,[6] R. C. Budhani,[7] Mintu Mondal,[8,9] John Jesudasan,[9] Pratap Raychaudhuri,[9] S. Caprara,[1] and L. Benfatto[1,*]



¿GV?

...even worse with non-linear *IV* fit. Why?

(finite size effects on RG do not work)

BKT and Inhomogeneities

$$\frac{R}{R_N} \sim \left(\frac{\xi_0}{\xi}\right)^2 \sim e^{-2b/\sqrt{t}}$$

Works only at T→ T^+_{BKT}

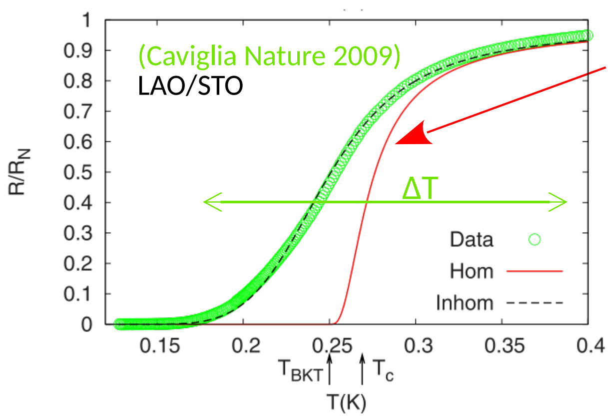
PHYSICAL REVIEW B **80**, 214506 (2009)

Broadening of the Berezinskii-Kosterlitz-Thouless superconducting transition by inhomogeneity and finite-size effects

L. Benfatto,[1,2] C. Castellani,[2] and T. Giamarchi[3]

(almost) Full R(T) range

$$\frac{R}{R_N} = \frac{1}{1+(\xi/\xi_0)^2}$$



BKT fluctuations

$$\frac{R}{R_N} = \frac{1}{1+\left(k\sinh\left(\frac{b}{\sqrt{t}}\right)\right)^2}$$

$$t = \frac{T - T_{BKT}}{T_{BKT}},$$

$$\boxed{b^2k^2 = R_N/R_c}$$

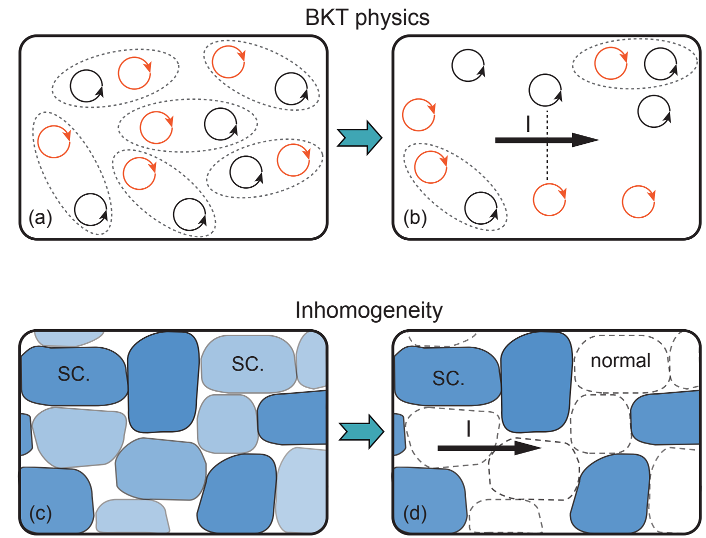
$$t_c \simeq 2.72 R_N/R_c \sim 0.01 \div 0.1$$

Cooper pair fluct

$$R_c = 16\hbar^2/e^2 = 65.6\,\mathrm{k}\Omega$$

$$b \sim \frac{\mu}{J_S}\sqrt{t_c}$$

$$k \sim 1,\ b \sim 0.15$$



Rule of thumb:

$$\frac{\Delta T}{T_c} \sim 1$$

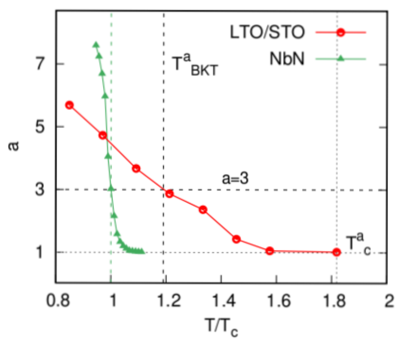
not θ fluctuations…

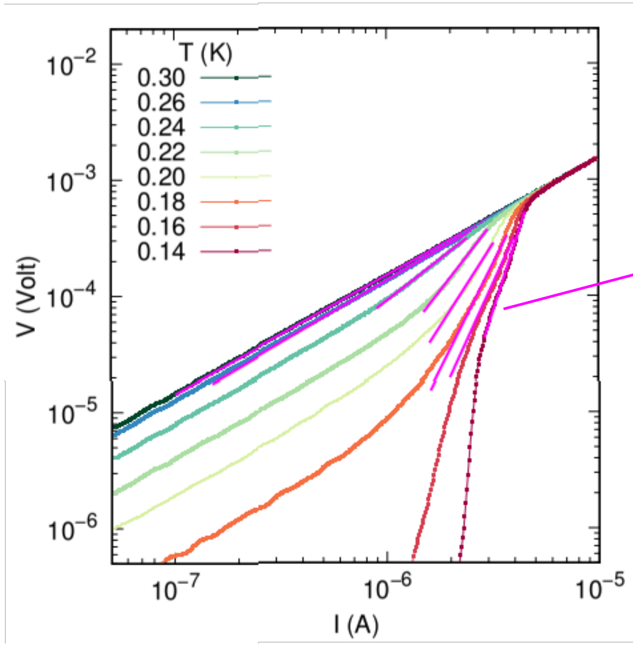
$$\frac{\Delta T}{T_c} < 1$$

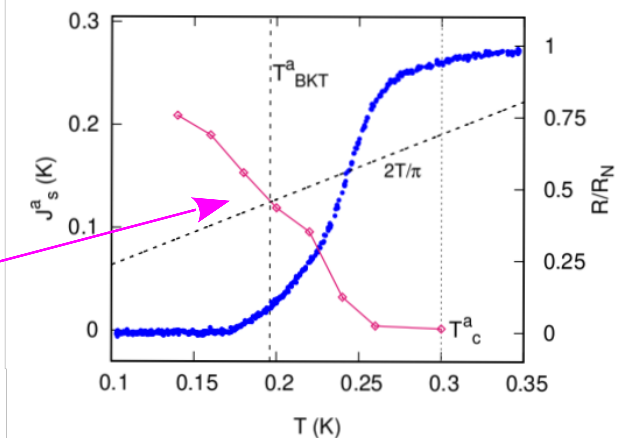
✓ but be careful

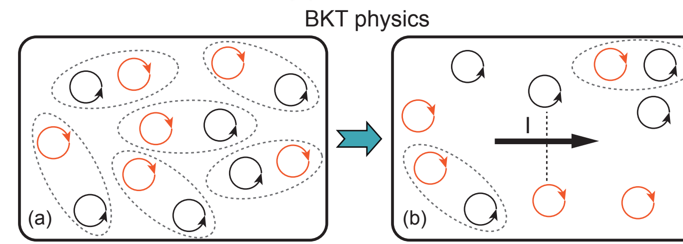
Finite size effects and **inhomogeneities**, **correlated disorder** can prevent the observation of BKT signatures

Observation of BKT signatures?

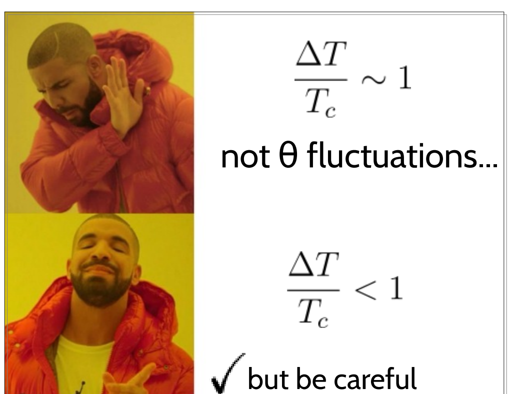








Rule of thumb:



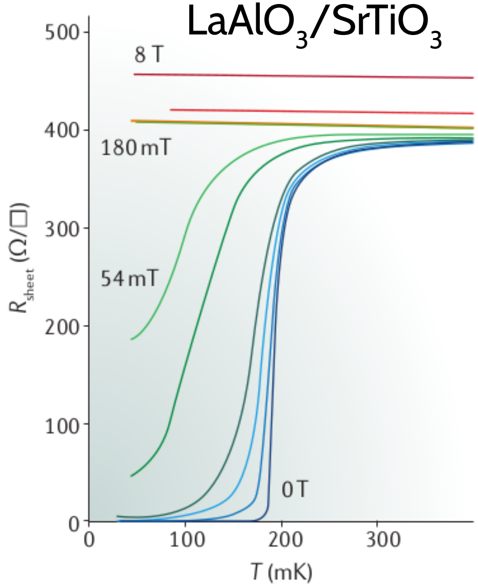
BKT $\underset{\not\Leftarrow}{\Rightarrow}$ non-linear *IV*

PHYSICAL REVIEW B **100**, 064506 (2019)

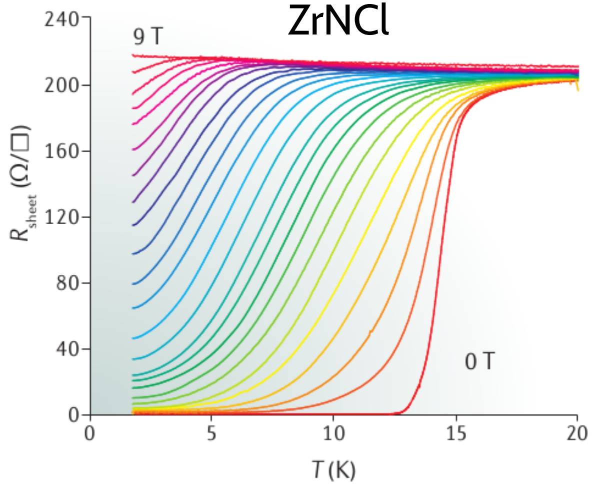
Nonlinear *I-V* characteristics of two-dimensional superconductors: Berezinskii-Kosterlitz-Thouless physics versus inhomogeneity

G. Venditti,[1] J. Biscaras,[2] S. Hurand,[3,4] N. Bergeal,[3,5] J. Lesueur,[3,5] A. Dogra,[6] R. C. Budhani,[7] Mintu Mondal,[8,9] John Jesudasan,[9] Pratap Raychaudhuri,[9] S. Caprara,[1] and L. Benfatto[1,*]

$LaAlO_3/SrTiO_3$

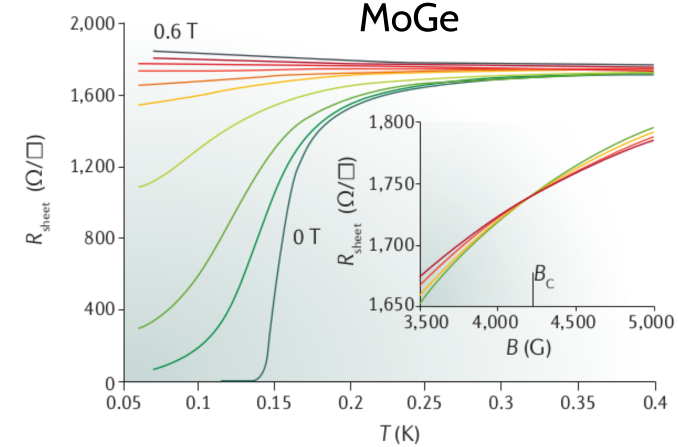


ZrNCl



Yu Saito et al, Nat. Rev. Mat. 2017

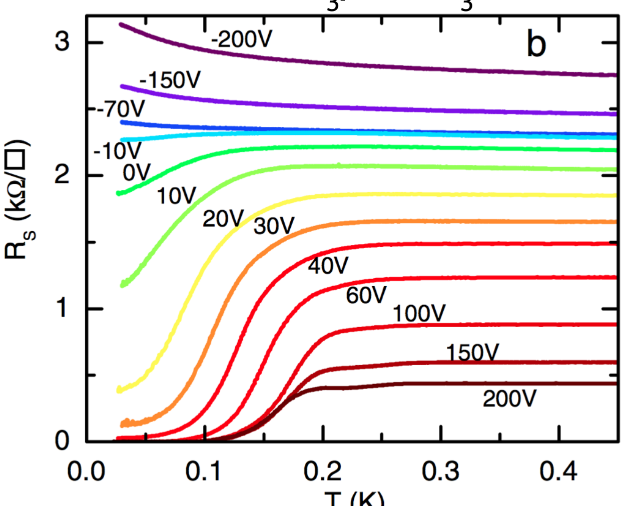
MoGe



¿LQ?

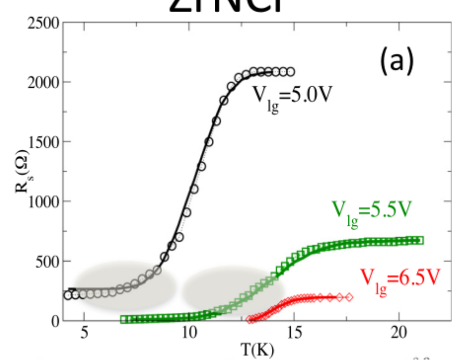
"quantum metal"(?!)
"failed SC" (?!)

$LaTiO_3/SrTiO_3$

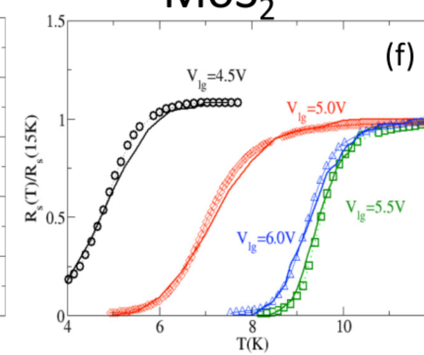


Biscaras et al, PRL 2012

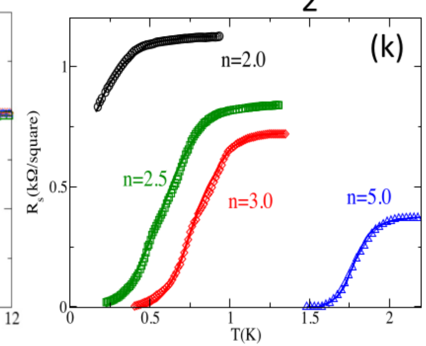
ZrNCl



MoS_2



$TiSe_2$

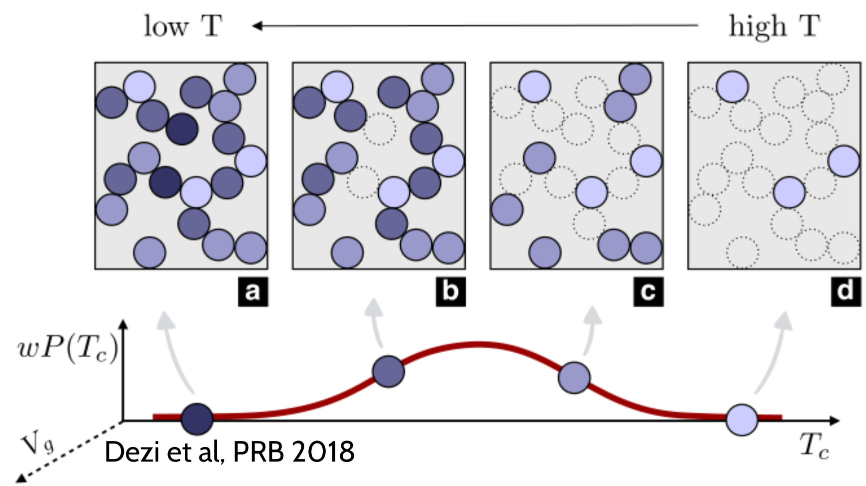


Dezi et al, PRB 2018

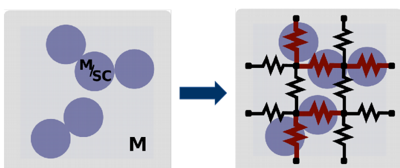
Inhomogeneities

Signatures of inhomogeneities and filamentarity:

- Broad transitions
    - Long tails
- Low T plateaux



Dezi et al, PRB 2018



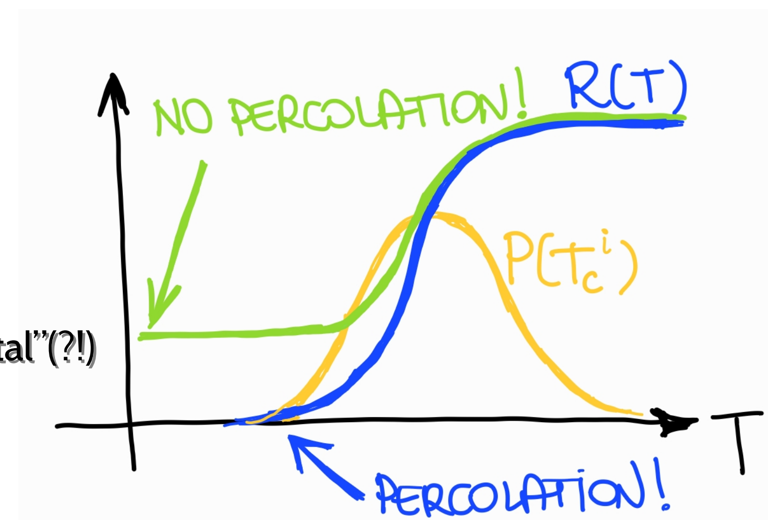
$$R_i = \begin{cases} R_n & \text{if } T > T_c^i, \\ 0 & \text{if } T \leq T_c^i \end{cases}$$

- No phase fluctuations
- Only inhomogeneities
- **Percolating transition**

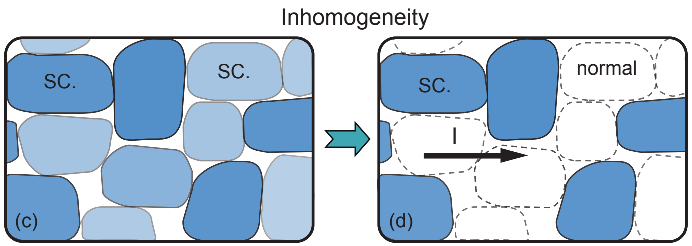
¡LQ?

"quantum metal"(?!)

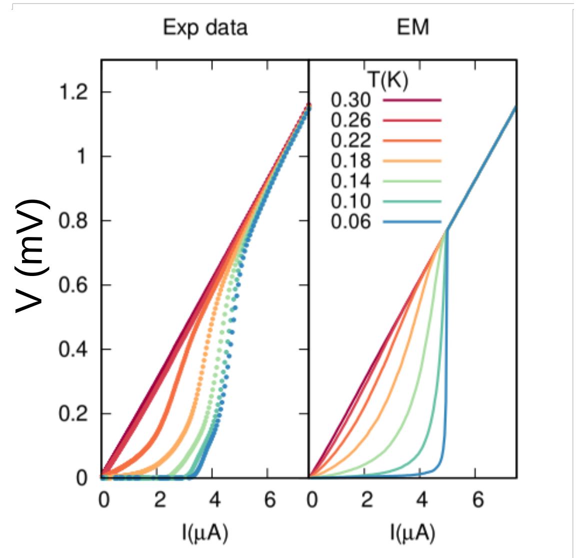
"failed SC" (?!)

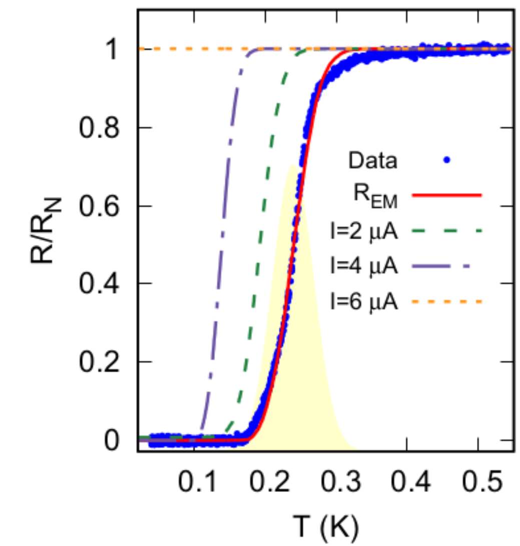


Inhomogeneities



$$R_i = \begin{cases} R_n \text{ if } T > T_c^i, \\ 0 \quad \text{if } T \leq T_c^i \text{ and } I \leq I_c^i \\ R_n \text{ if } T \leq T_c^i \text{ and } I > I_c^i \end{cases}$$





Non-linear IV produced by inhomogeneities!

BKT $\rightleftarrows$ non-linear *IV*

- No phase fluctuations
- Only inhomogeneities
- **Percolating transition**

PHYSICAL REVIEW B **100**, 064506 (2019)

Nonlinear *I*-*V* characteristics of two-dimensional superconductors: Berezinskii-Kosterlitz-Thouless physics versus inhomogeneity

G. Venditti,[1] J. Biscaras,[2] S. Hurand,[3,4] N. Bergeal,[3,5] J. Lesueur,[3,5] A. Dogra,[6] R. C. Budhani,[7] Mintu Mondal,[8,9] John Jesudasan,[9] Pratap Raychaudhuri,[9] S. Caprara,[1] and L. Benfatto[1,*]

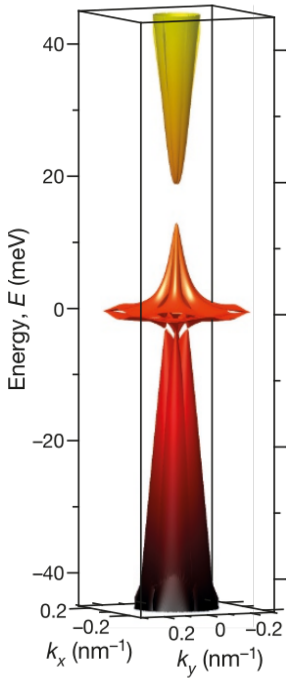
¿GV?

BKT in TBG?

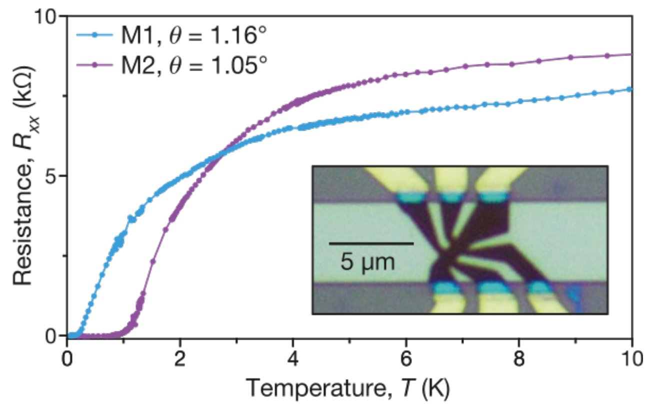
Cao et al., Nature 2018

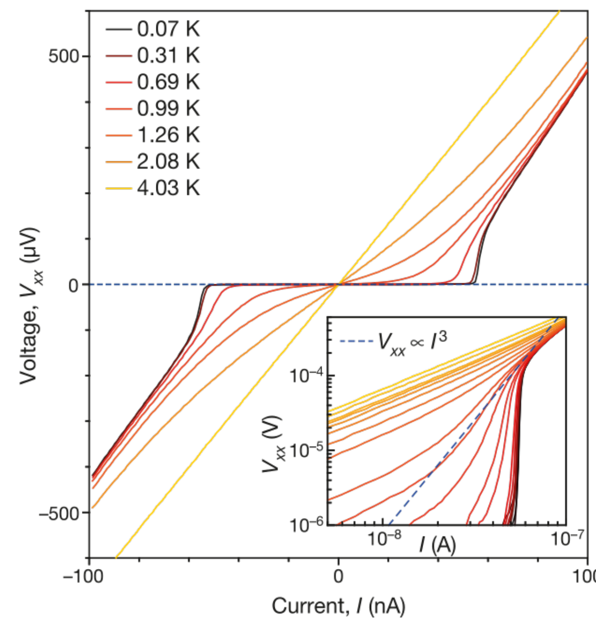
Unconventional superconductivity in magic-angle graphene superlattices

Yuan Cao[1], Valla Fatemi[1], Shiang Fang[2], Kenji Watanabe[3], Takashi Taniguchi[3], Efthimios Kaxiras[2,4] & Pablo Jarillo-Herrero[1]



BKT signatures?





- Topological flat bands
- Moiré potential
- (not very) Normal state

To summarize...

- BKT summary:
  - ✓ U(1) symmetry but quasi-long-range order
  - ✓ $\delta\theta$, $\delta\Psi_0$, d-wave, spin-wave, ..., change J_S and T_{BKT}

    but $J_S = 2T_{BKT}/\pi$

- It's hard to see BKT signatures in real 2d SC

  screening currents, finite size effects, correlated disorder/inhomogeneities, ...

- Non-linear *IVs* <u>do not</u> imply BKT

  → Why J_S from non-linear IV are smeared out? ¡GV?

¡LR?

Role of vortex **pinning**?
→ Change of dynamics...
(another Flat Club topic?)

¡GV?

→ What if we mess up with symmetries and v-av pairs?

...Moiré potential? ...Majorana zero modes?

Aknowledgements

DIPARTIMENTO DI FISICA
SAPIENZA
UNIVERSITÀ DI ROMA

Sergio Caprara

Marco Grilli

Bernard Van Heck

Lara Benfatto

PAUL SCHERRER INSTITUT
PSI

Ilaria Maccari

DQMP Department of Quantum Matter Physics

Louk Rademaker

Christophe Berthod

Flat and Strange Quantum Matter group