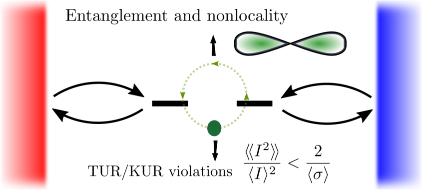
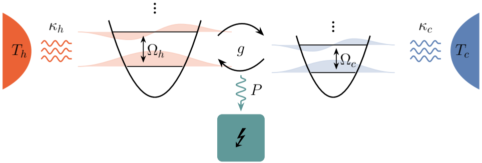
Nonclassical behavior in open quantum systems:

wave-particle duality, entanglement, and thermo-kinetic uncertainty relations





arXiv:2212.03835

arXiv:2303.09244

Kacper Prech, Marcelo Janovitch, Matteo Brunelli, Patrick P. Potts

Swiss National Science Foundation

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The Team

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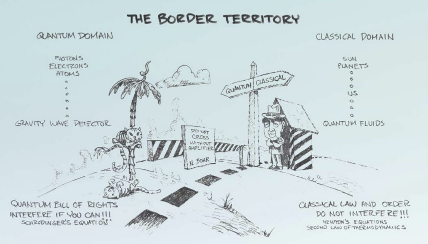
Nonclassical behavior



Zurek, in *Quantum Decoherence*. Prog. Math. Phys. **48**, (2006)

Motivation

- Fundamental: Understand the difference between quantum and classical
- Practical: Exploit quantum effects for technologies

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Types of nonclassical behavior

Fundamental nonclassicality

- Bell nonlocality - Bell inequalities
- Contextuality - contextuality inequalities

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Technological nonclassicality

- Cryptography, computing
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Nonclassical scaling

- Quantum sensing - Heisenberg scaling vs standard quantum limit
- Quantum computing - number of qubits vs number of bits

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Nonclassical scaling

- Quantum sensing - Heisenberg scaling vs standard quantum limit
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Ruling out classical models

- Single classical model - QED vs classical ED
- Ruling out class of models - Thermodynamic uncertainty relations

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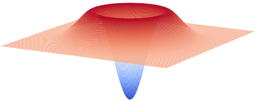
Ruling out classical models

Observations cannot be reproduced by adequate classical model(s)

Example - quantum optics

Experimental observations in quantum optics may not be explained by classical electro-dynamics

Mandel, Phys. Scr. **1986**, 34

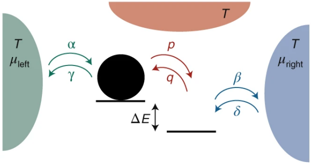


Example - thermodynamic uncertainty relation

$$\frac{2\langle I\rangle^2}{\langle\langle I^2\rangle\rangle\langle\sigma\rangle} \leq 1$$

Violation rules out classical Markovian theories

Horowitz, Gingrich, Nat. Phys. **16**, 15 (2020)



Insight into the quantum-to-classical transition

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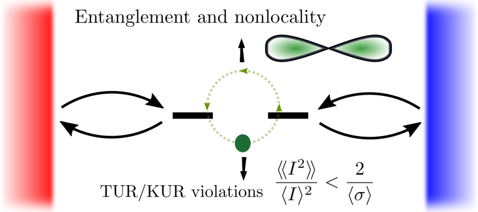
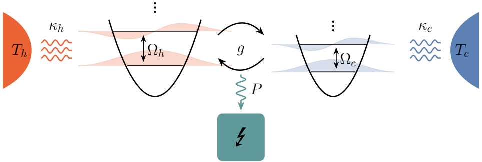
Nonclassical behavior in quantum transport

Nonclassical behavior in quantum transport





- Fermionic double quantum dot
- Manifestations of coherence
- Compare to classical rate equation

- Bosonic quantum heat engine
- Wave-particle duality
- Compare to wave/particle model

When can transport be described using classical models?

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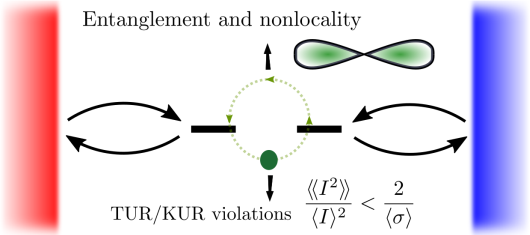
Entanglement and thermo-kinetic uncertainty relations in coherent mesoscopic transport



arXiv:2212.03835 (accepted in Phys. Rev. Research)

K. Prech, P. Johansson, E. Nyholm, G. T. Landi, C. Verdozzi, P. Samuelsson, P. P. Potts

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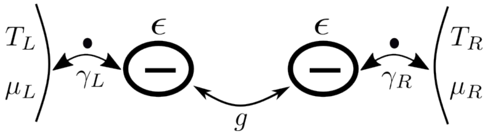
The system

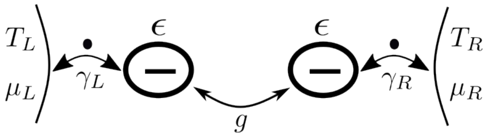


- g: inter-dot tunneling
- ϵ: on-site energy
- γ_α: system-bath couplings

$$\hat{H} = \epsilon \left(\hat{c}_R^\dagger \hat{c}_R + \hat{c}_L^\dagger \hat{c}_L \right) + g \left(\hat{c}_L^\dagger \hat{c}_R + \hat{c}_R^\dagger \hat{c}_L \right)$$

A voltage ($\mu_L - \mu_R = eV$) or temperature bias induces a charge current

The system



- g: inter-dot tunneling
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- γ_α: system-bath couplings

Manifestations of coherence

- Entanglement - Bohr Brask, Haack, Brunner, Huber, New J. Phys. **17**, 113029 (2015)
- Violations of TUR - Ptaszyński, Phys. Rev. B **98**, 085425 (2018)
- Violations of KUR

Goal of our work: understand when these different manifestations of coherence appear

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Entanglement

There is entanglement between Alice (A) and Bob (B) iff

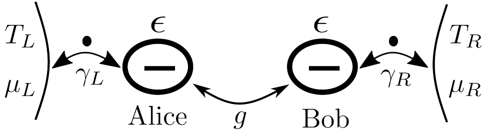
$$\hat{\rho} \neq \sum_i p_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i$$

There is entanglement between Left (L) and Right (R) dot iff

$$\hat{\rho} \neq \sum_i p_i \hat{\rho}_L^i \otimes \hat{\rho}_R^i$$

DQD steady state

$$\hat{\rho} = \begin{pmatrix} p_0 & 0 & 0 & 0 \\ 0 & p_L & \alpha & 0 \\ 0 & \alpha^* & p_R & 0 \\ 0 & 0 & 0 & p_D \end{pmatrix}$$



$\Rightarrow$ when $|\alpha|^2 > p_0 p_D$, then there is entanglement between dots

Thermodynamic uncertainty relations

$$\mathcal{Q}_T \equiv \frac{2\langle I\rangle^2}{\langle\langle I^2\rangle\rangle\langle\sigma\rangle} \leq 1$$

Holds for classical Markovian models.

- Average current: $\langle I\rangle$
- Current noise: $\langle\langle I^2\rangle\rangle \equiv \int_{-\infty}^{\infty} dt[\langle I(t)I(0)\rangle - \langle I\rangle^2]$
- Entropy production: $\langle\sigma\rangle = -\frac{J_c}{T_c} - \frac{J_h}{T_h}$

Entropy production (dissipation) upper bounds the signal-to-noise ratio!

Horowitz, Gingrich, Nat. Phys. **16**, 15 (2020)

Thermodynamic uncertainty relations

$$\mathcal{Q}_T \equiv \frac{2\langle I\rangle^2}{\langle\langle I^2\rangle\rangle\langle\sigma\rangle} \leq 1$$

Holds for classical Markovian models.

Kinetic uncertainty relation

$$\mathcal{Q}_K \equiv \frac{\langle I\rangle^2}{\langle\langle I^2\rangle\rangle\langle\mathcal{A}\rangle} \leq 1$$

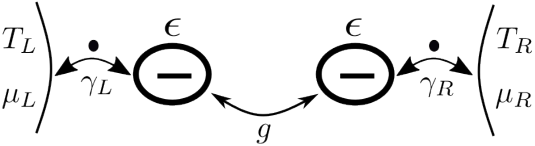
$\langle\mathcal{A}\rangle$: dynamical activity

Terlizzi, Baiesi, J. Phys. A **52**, 02LT03 (2018)

$$\mathcal{Q}_{TK} \equiv \frac{\langle I\rangle^2}{\langle\langle I^2\rangle\rangle} f(\langle\mathcal{A}\rangle, \langle\sigma\rangle) \leq 1$$

Vo, Vu, Hasegawa, J. Phys. A **55**, 405004 (2022)

Models



NEGFs

- Exact for noninteracting electrons
- Equivalent to Landauer-Büttiker scattering theory

$$\langle I \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{T}(\omega) \left[f_L(\omega) - f_R(\omega) \right]$$

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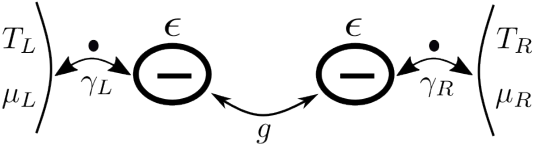
Models

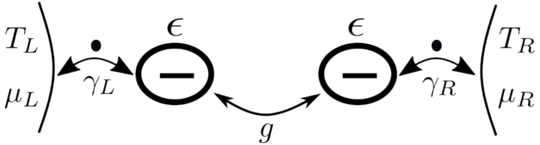


Local master equation

$$\partial_t \hat{\rho} = -i\big[\hat{H}, \hat{\rho}\big] + \sum_{\alpha=h,c} f_\alpha \gamma_\alpha \mathcal{D}[\hat{c}_\alpha^\dagger]\hat{\rho} + (1 - f_\alpha)\gamma_\alpha \mathcal{D}[\hat{c}_\alpha]\hat{\rho}$$

$\mathcal{D}[\hat{A}]\hat{\rho} = \hat{A}\hat{\rho}\hat{A}^\dagger - \frac{1}{2}\{\hat{A}^\dagger \hat{A}, \hat{\rho}\}$ $f_\alpha = \frac{1}{e^{(\epsilon-\mu_\alpha)/k_B T_\alpha} + 1}$

Models

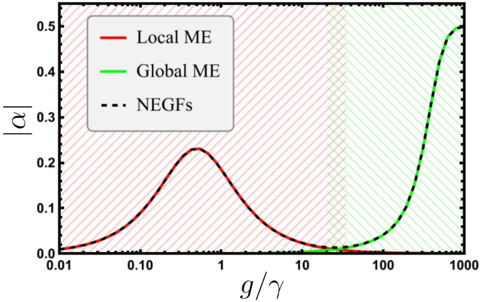


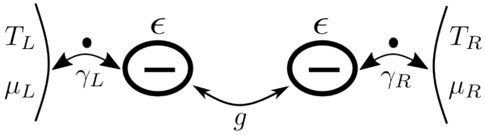
Classical model

$$\partial_t \vec{p} = W \vec{p}$$

Inter-dot rate: $W_{LR} = \frac{4g^2}{\gamma_L + \gamma_R}$, all other rates same as in local master equation

Coherence





Peak in coherence

- Coherent tunneling results in Rabi oscillations between left and right dot
- Rabi oscillations interrupted by classical jumps in and out of the system
- For small g/γ, Rabi-oscillations interrupted after short time
- For large g/γ, random phase is acquired before a classical jump

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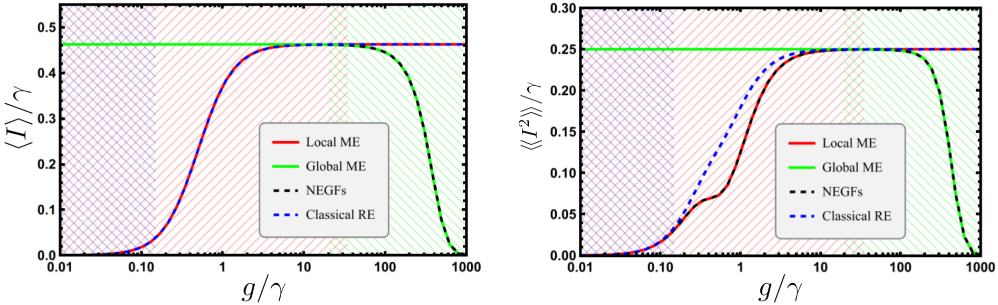
Current and noise



$$\langle\langle I^2 \rangle\rangle \equiv \int_{-\infty}^{\infty} dt [\langle I(t) I(0) \rangle - \langle I \rangle^2]$$

- Average current reproduced by classical model (when local ME is valid)
- Fluctuations suppressed where the peak in coherence is

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Manifestations of coherence

- Entanglement: static manifestation of coherence
- TUR/KUR violations: dynamic manifestations of coherence

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Manifestations of coherence

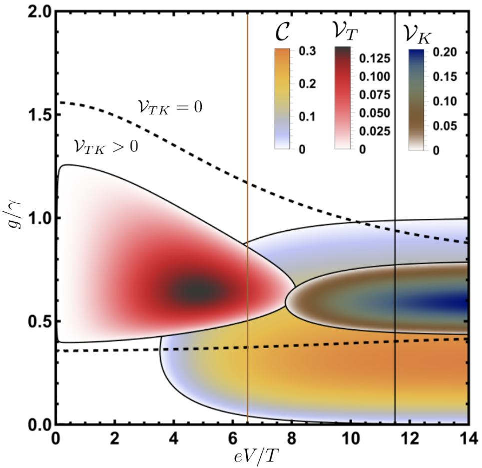
Concurrence

$$\mathcal{C} = \text{Max}\left\{2|\alpha| - 2\sqrt{p_0 p_D}, 0\right\}$$

T/KUR-violations

$$\mathcal{V}_j = \max\{\mathcal{Q}_j - 1, 0\}$$

- Manifestations at $g \simeq \gamma$
- TUR-violations at small dissipation
- KUR-violations at large dissipation



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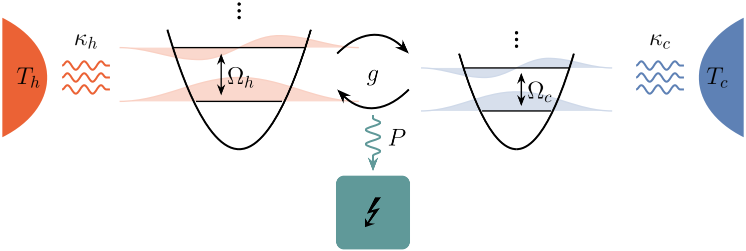
The wave-particle duality in a quantum heat engine



arXiv:2303.09244

Marcelo Janovitch, Matteo Brunelli, Patrick P. Potts

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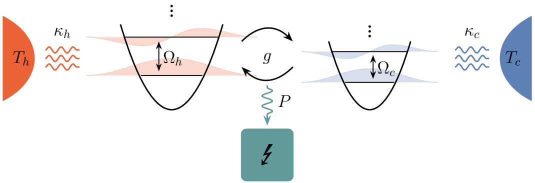
The quantum model



$$\hat{H}(t) = \Omega_h \hat{a}_h^\dagger \hat{a}_h + \Omega_c \hat{a}_c^\dagger \hat{a}_c + g\left(\hat{a}_h^\dagger \hat{a}_c e^{-i\Delta t} + \hat{a}_c^\dagger \hat{a}_h e^{i\Delta t}\right) \qquad \Delta = \Omega_h - \Omega_c$$

Local master equation

$$\partial_t \hat{\rho} = -i[\hat{H}(t), \hat{\rho}] + \sum_{\alpha=h,c} \bar{n}_\alpha \kappa_\alpha \mathcal{D}[\hat{a}_\alpha^\dagger]\hat{\rho} + (\bar{n}_\alpha + 1)\kappa_\alpha \mathcal{D}[\hat{a}_\alpha]\hat{\rho}$$

$\mathcal{D}[\hat{A}]\hat{\rho} = \hat{A}\hat{\rho}\hat{A}^\dagger - \frac{1}{2}\{\hat{A}^\dagger\hat{A}, \hat{\rho}\}$ $\qquad$ $\bar{n}_\alpha = \frac{1}{e^{\Omega_\alpha/k_B T_\alpha} - 1}$

Hofer, Souquet, Clerk, PRB **93**, 041418(R) (2016); Kosloff, J. Chem. Phys. **80**, 1625 (1984)

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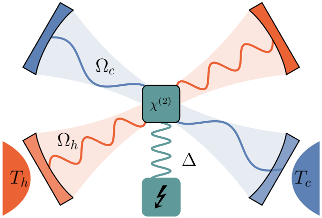
The wave model



Quantum Langevin equations (Heisenberg picture)

$$\partial_t \hat{a}_h = -\left(i\Omega_h + \frac{\kappa_h}{2}\right)\hat{a}_h - ig\hat{a}_c e^{-it\Delta} - \sqrt{\kappa_h}\hat{\xi}_h$$

$$\partial_t \hat{a}_c = -\left(i\Omega_c + \frac{\kappa_c}{2}\right)\hat{a}_c - ig\hat{a}_h e^{+it\Delta} - \sqrt{\kappa_c}\hat{\xi}_c$$

Quantum white noise: $\langle \hat{\xi}^\dagger_\alpha(t')\hat{\xi}_\beta(t)\rangle_q = \bar{n}_\alpha \delta_{\alpha\beta}\delta(t'-t)$, $[\hat{\xi}_\alpha(t'), \hat{\xi}^\dagger_\beta(t)] = \delta_{\alpha\beta}\delta(t'-t)$

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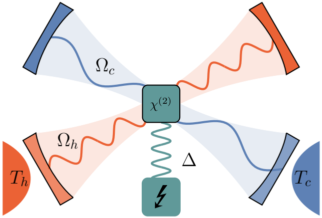
The wave model



Classical Langevin equations

$$\partial_t A_h = -\left(i\Omega_h + \frac{\kappa_h}{2}\right) A_h - igA_c e^{-it\Delta} - \sqrt{\kappa_h}\xi_h$$

$$\partial_t A_c = -\left(i\Omega_c + \frac{\kappa_c}{2}\right) A_c - igA_h e^{+it\Delta} - \sqrt{\kappa_c}\xi_c$$

Classical white noise: $\langle \xi_\alpha^*(t')\xi_\beta(t)\rangle_w = \bar{n}_\alpha \delta_{\alpha\beta}\delta(t'-t)$

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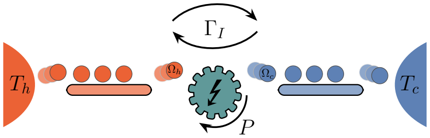
The particle model



Classical rate equation

$$\begin{aligned}\partial_t p_{n_h,n_c} &= \kappa_h(\bar{n}_h+1)(n_h+1)p_{n_h+1,n_c} + \kappa_h\bar{n}_h n_h p_{n_h-1,n_c} \\ &+ \kappa_c(\bar{n}_c+1)(n_c+1)p_{n_h,n_c+1} + \kappa_c\bar{n}_c n_c p_{n_h,n_c-1} \\ &+ \Gamma_I(n_h+1)n_c p_{n_h+1,n_c-1} + \Gamma_I(n_c+1)n_h p_{n_h-1,n_c+1} - \Gamma^0_{n_h,n_c}p_{n_h,n_c}\end{aligned}$$

- Rates between system and bath same as in master equation
- Intra-system hopping: $\Gamma_I = \frac{4g^2}{\kappa_c+\kappa_h}$

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Average power

$$\langle P \rangle_q = -\langle \partial_t \hat{H}(t) \rangle_q$$

$$\langle P \rangle_q = \langle P \rangle_w = \langle P \rangle_p = \frac{4g^2 \kappa_h \kappa_c \Delta(\bar{n}_h - \bar{n}_c)}{(4g^2 + \kappa_h \kappa_c)(\kappa_h + \kappa_c)}$$

$$= \Delta(\bar{n}_h - \bar{n}_c)(\kappa_h^{-1} + \kappa_c^{-1} + \Gamma_I^{-1})^{-1}$$

- The classical models reproduce the average power
- Resembles series addition of three conductances

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Noise

$$\langle\langle P^2 \rangle\rangle = \int_{-\infty}^{\infty} dt \left[\langle P(t) P(0) \rangle - \langle P \rangle^2 \right]$$

$$\begin{aligned}
\langle\langle P^2 \rangle\rangle_q &= \mathcal{E}[\bar{n}_h(\bar{n}_h + 1) + \bar{n}_c(\bar{n}_c + 1)] - \mathcal{S}(\bar{n}_h - \bar{n}_c)^2, \\
\langle\langle P^2 \rangle\rangle_w &= \mathcal{E}(\bar{n}_h^2 + \bar{n}_c^2) - \mathcal{S}(\bar{n}_h - \bar{n}_c)^2, \\
\langle\langle P^2 \rangle\rangle_p &= \mathcal{E}[\bar{n}_h(\bar{n}_h + 1) + \bar{n}_c(\bar{n}_c + 1)] - \mathcal{S}_p(\bar{n}_h - \bar{n}_c)^2,
\end{aligned}$$

$$\mathcal{E} = \tfrac{\langle P \rangle \Delta}{\bar{n}_h - \bar{n}_c} \qquad\qquad \mathcal{S}_p \geq \mathcal{S}$$

- The wave model reproduces shot noise but not vacuum fluctuations
- The particle model reproduces equilibrium noise but exhibits reduced bunching

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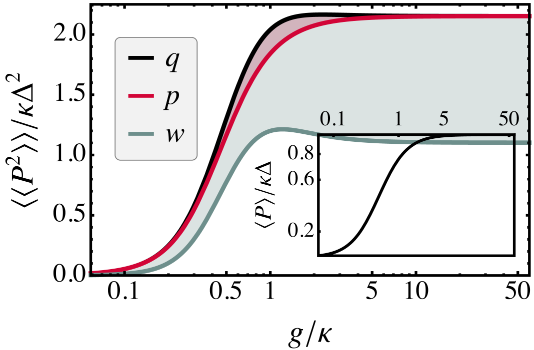
Wave-particle duality



- Particle model valid for $g \gg \kappa$ and $g \ll \kappa$
- Wave model valid for high temperatures
- Generally, neither wave nor particle descriptions adequate

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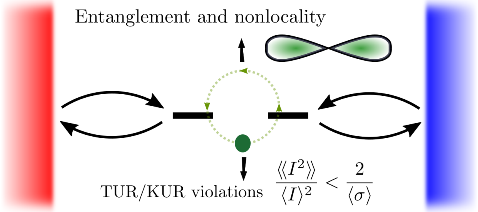
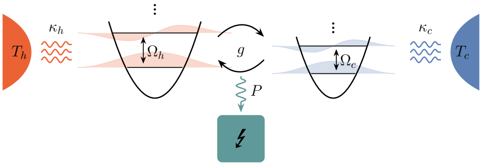
Conclusions and outlook



arXiv:2212.03835

- Static and dynamic manifestations of coherence for $g \simeq \gamma$
- Model based on classical waves?



arXiv:2303.09244

- Neither waves nor particles reproduce noise
- Can we exploit this insight?

Apply classical "wave" and "particle" models to other setups (e.g. spins, qubits)

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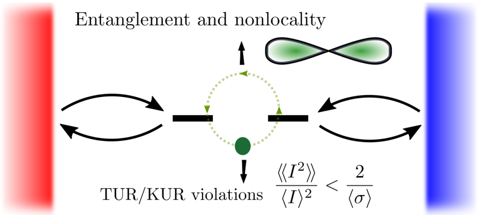
Conclusions and outlook



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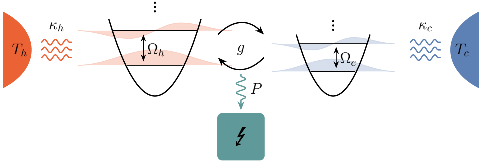

arXiv:2303.09244

Tutorial on Fluctuations

Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics

Landi, Kewming, Mitchison, Potts

arXiv:2303.04270