
# Optical conductivity of strange metals

Christophe Berthod

FLAT Club

Geneva, 30/09/2022

[1] arXiv:2204.10284
[2] arXiv:2205.00899 [PRB **106**, 054515 (2022)]
[3] arXiv:2205.04030

# Outline

1. Optical spectroscopy
    - How to measure ?
    - How to interpret ?
    - How to calculate ?
2. Strange metals
    - Linear resistivity, so what ?
    - Strange versus bad metals
    - Generic model of Planckian dissipation
3. Today's menu

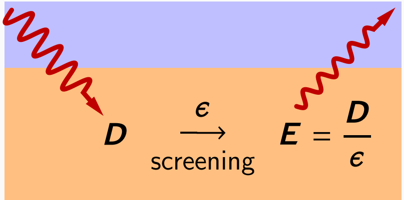
[3] arXiv:2205.04030
[2] arXiv:2205.00899 [PRB **106**, 054515 (2022)]
[1] arXiv:2204.10284

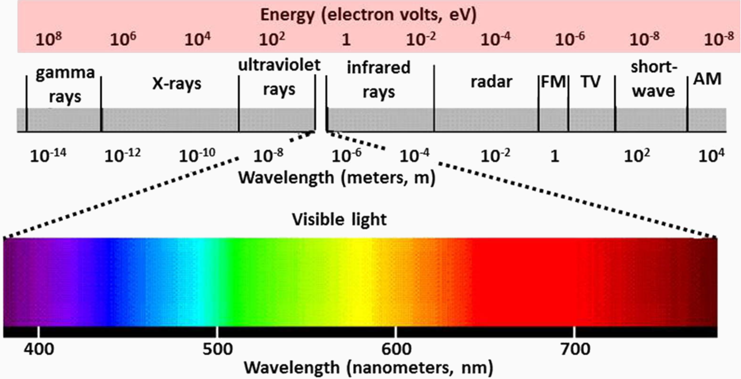
# Optical spectroscopy (infrared)

Target of the experiment $\epsilon(\boldsymbol{q}, \omega)$ the dielectric function

"Screening is one of the most important concepts in many-body theory"

Gerald D. Mahan




Infrared photons $\epsilon(\omega) \equiv \epsilon(\boldsymbol{q} = \boldsymbol{0}, \omega)$

# Optical measurements

## Reflectivity

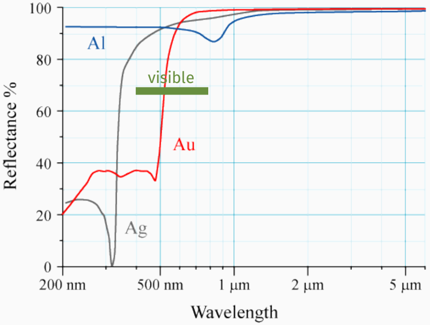
$$R = \left| \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}} \right|^2$$

$+$

Kramers–Kronig

$=$

$$\epsilon_1(\omega) + i\epsilon_2(\omega)$$




$R \sim 1$

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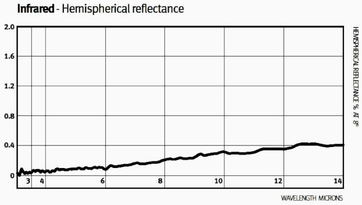




$R \sim 0$

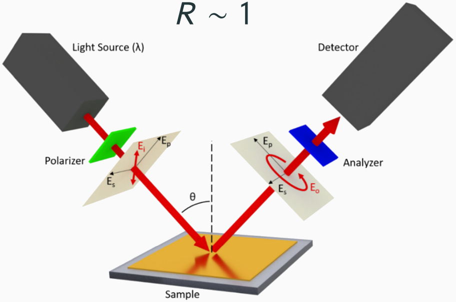
## Ellipsometry

$+$

Fresnel equations

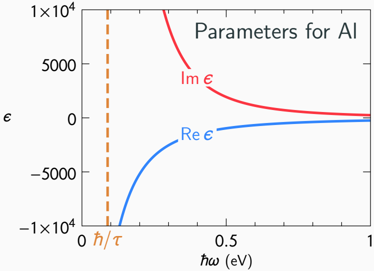
$=$

$$\epsilon_1(\omega) + i\epsilon_2(\omega)$$



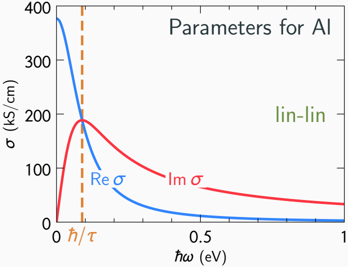
# Qualitative expectations for a metal

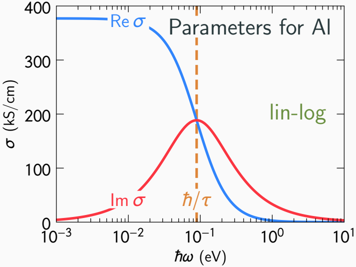
**Drude model** $$\epsilon(\omega) = 1 - \frac{ne^2/\epsilon_0 m}{\omega^2 + 1/\tau^2} + \frac{i}{\omega\tau}\frac{ne^2/\epsilon_0 m}{\omega^2 + 1/\tau^2}$$

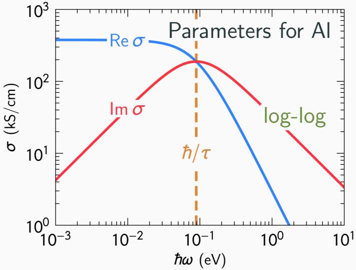



Conductivity $$\sigma(\omega) = i\epsilon_0\omega[1 - \epsilon(\omega)]$$

**Drude model** $$\sigma(\omega) = \frac{ne^2/m}{-i\omega + 1/\tau}$$



# Computing the conductivity

## Kubo formula

$$\sigma(\omega) = \frac{i}{\omega}\left[C_{J_xJ_x}(\omega) + \frac{ne^2}{m}\right], \qquad C_{J_xJ_x}(\omega) = -\frac{i}{\hbar}\int_0^\infty dt\, e^{i\omega t}\langle[J_x(t), J_x(0)]\rangle$$

## Local limit (no vertex corrections)

$$\sigma(\omega) = \frac{ie^2}{\omega}\sum_{\boldsymbol{k}\sigma} v_{\boldsymbol{k}x}^2 \int_{-\infty}^{\infty} d\varepsilon_1 d\varepsilon_2\, \frac{f(\varepsilon_1) - f(\varepsilon_2)}{\hbar\omega + \varepsilon_1 - \varepsilon_2 + i0} A(\boldsymbol{k}, \varepsilon_1)A(\boldsymbol{k}, \varepsilon_2)$$

$$v_{\boldsymbol{k}x} = \frac{1}{\hbar}\frac{\partial \varepsilon_{\boldsymbol{k}}}{\partial k_x}$$

$$A(\boldsymbol{k}, \varepsilon) = \frac{-\Sigma_2(\varepsilon)/\pi}{[\varepsilon - \varepsilon_{\boldsymbol{k}} - \Sigma_1(\varepsilon)]^2 + [\Sigma_2(\varepsilon)]^2}$$

$$\Sigma(\varepsilon) = \Sigma_1(\varepsilon) + i\Sigma_2(\varepsilon)$$

**Ingredients of the calculation**

| | |
|---|---|
| $\varepsilon_{\boldsymbol{k}}$ | one-particle dispersion |
| $\Sigma(\varepsilon)$ | one-particle self-energy |

# $\epsilon_\infty$ — Separating high- and low-energy transitions

Can we compare $\sigma_{\text{exp}}(\omega) = i\epsilon_0\omega[1 - \epsilon(\omega)]$ with $\sigma_{\text{theo}}(\omega)$ ?

No — $\sigma_{\text{theo}}$ contains only low-energy transitions, while $\sigma_{\text{exp}}$ contains all transitions.

We must separate high- and low-energy transitions

$$\epsilon(\omega) = 1 + \epsilon_{\text{low}}(\omega) + \epsilon_{\text{high}}(\omega)$$

and subtract the high-energy transitions

$$\sigma_{\text{low}}(\omega) = i\epsilon_0\omega\left\{1 - [\epsilon(\omega) - \epsilon_{\text{high}}(\omega)]\right\}$$

If the high-energy transitions are well separated

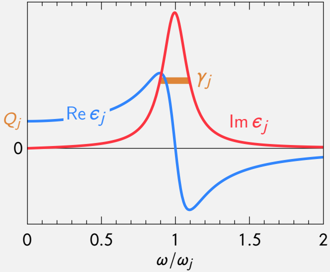
$$\sigma_{\text{low}}(\omega) \approx i\epsilon_0\omega[\underbrace{1 + \epsilon_{\text{high}}(0)}_{\epsilon_\infty} - \epsilon(\omega)]$$

### Standard conversion formula

$$\sigma(\omega) = i\epsilon_0\omega[\epsilon_\infty - \epsilon(\omega)]$$

### Lorentz oscillator

$$\epsilon_j(\omega) = \frac{Q_j\omega_j^2}{\omega_j^2 - \omega(\omega + i\gamma_j)}$$



# Extended Drude "model"

Drude conductivity

$$\sigma_{\text{Drude}}(\omega) = \frac{ne^2/m}{-i\omega + 1/\tau}$$

Sum rule

$$\frac{2}{\pi}\int_0^\infty d\omega\, \text{Re}\,\sigma_{\text{Drude}}(\omega) = \frac{ne^2}{m}$$

Ohm's law $\sigma = \frac{J}{E} = \frac{nev}{E}$

Newton's law $\dot{v} = \frac{eE}{m} - \frac{v}{\tau}$

$$(-i\omega + 1/\tau)v = \frac{eE}{m}$$

Extended Drude model

$$\sigma(\omega) = \frac{\epsilon_0\omega_p^2}{-i\omega m^*(\omega)/m + 1/\tau(\omega)}$$

Sum rule

$$\epsilon_0\omega_p^2 = \frac{2}{\pi}\int_0^\infty d\omega\, \text{Re}\,\sigma(\omega)$$

$$m \to m^*(\omega) \qquad \dot{v} \to \dot{v}\frac{m^*(\omega)}{m}$$

$$\frac{1}{\tau(\omega)} = \text{Re}\,\epsilon_0\omega_p^2\frac{1}{\sigma(\omega)}$$

$$\frac{m^*(\omega)}{m} = \text{Im}\,\frac{1}{-\omega}\epsilon_0\omega_p^2\frac{1}{\sigma(\omega)}$$

Reflectivity & ellipsometry
(+ know-how)

$$\downarrow$$

$$\epsilon(\omega)$$

$$\downarrow$$

$$\sigma(\omega) = i\epsilon_0\omega[\epsilon_\infty - \epsilon(\omega)]$$

$$\downarrow$$

$$\frac{1}{\tau(\omega)} = \epsilon_0\omega_p^2 \operatorname{Re}\frac{1}{\sigma(\omega)}$$

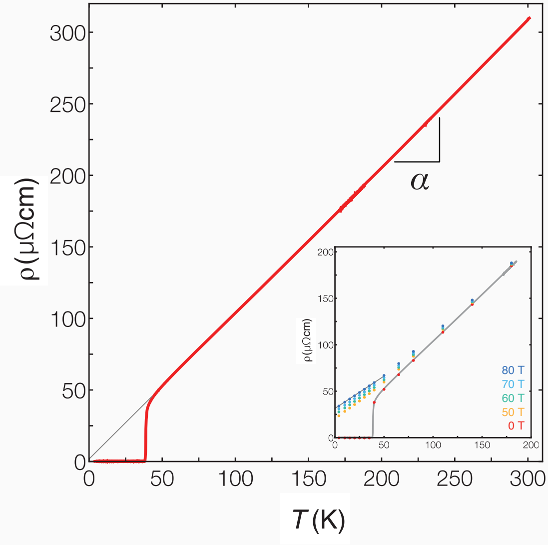
$$\frac{m^*(\omega)}{m} = \epsilon_0\omega_p^2 \operatorname{Im}\frac{-1}{\omega\sigma(\omega)}$$

## Linear resistivity

Observed in

high-$T_c$ cuprates,
some Fe-based superconductors,
some heavy-fermion materials,
twisted bilayer graphene,
etc...

$La_{2-x}Sr_xCuO_4$ @ $x = 0.19$



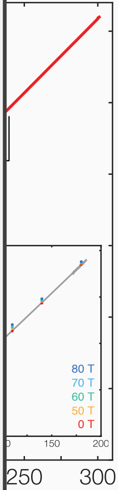

Giraldo-Gallo *et al.*, Science **361**, 479 (2018)

## Linear re

.19

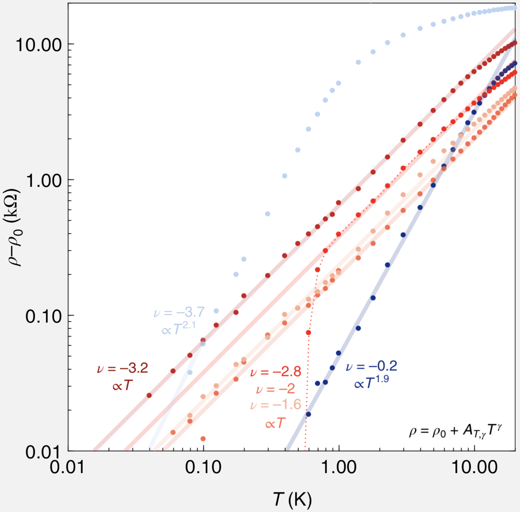
Obser

high-$T_c$ c
some Fe-based s
some heavy-fer
twisted bilay
etc




479 (2018)

### Twisted bilayer graphene



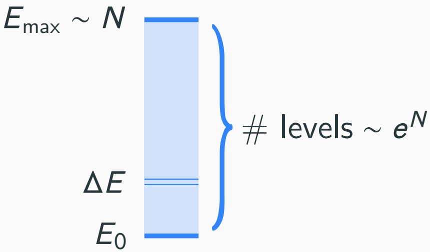

Jaoui *et al.*, Nat. Phys. **18**, 633 (2022)

# Resistivity and many-body spectrum

Because the energy is extensive, a system of size $N$ has maximum energy of order $N$.

How are the $\sim e^N$ energy levels distributed ?

For independent particles, the inter-level spacing at low-energy is $\Delta E \sim 1/N$.



# Resistivity and many-body spectrum

Because the energy is extensive, a system of size $E_{max} \sim N$

$N$

H

Fo

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```
N = 10; M = 5; ϵ = Range[N] / N; Histogram[(Plus @@ ϵ[[#]] &) /@ Permutations[Range[N], {M}]]
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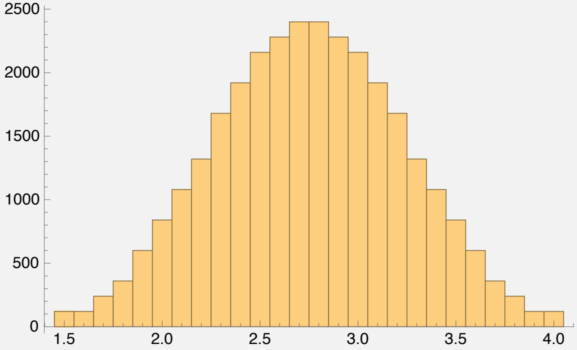

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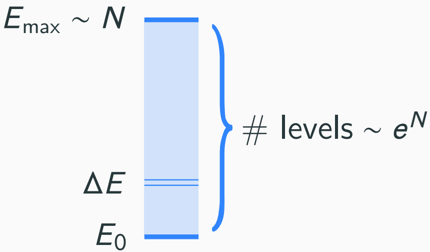
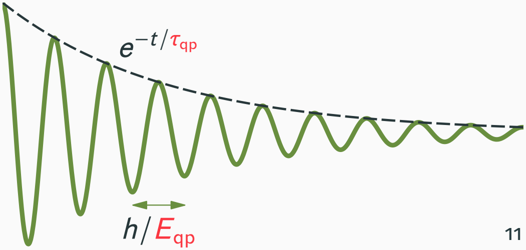



For Landau quasiparticles, $\Delta E \sim 1/N$ as well.

Pauli-limited scattering rate

$$\frac{\hbar}{\tau_{qp}} \sim U^2 N(E_F)\left(\frac{k_B T}{E_F}\right)^2 \ll E_{qp} \sim k_B T$$



## How fast can a quantum system equilibrate ?

Heisenberg uncertainty principle

$$\Delta E \cdot \Delta t \gtrsim \hbar$$

For a quantum system in equilibrium at temperature $T$

$$\Delta E \sim k_B T \quad \Rightarrow \quad \Delta t \gtrsim \frac{\hbar}{k_B T}$$

$8 \times 10^{-12}$ second at 1 Kelvin

## If transport times coincide with equilibration times

$$\tau = \alpha \frac{\hbar}{k_B T}, \qquad \alpha = \mathcal{O}(1) \quad \Rightarrow \quad \rho \propto \frac{1}{\tau} \propto T$$

# The good, the bad, and the strange

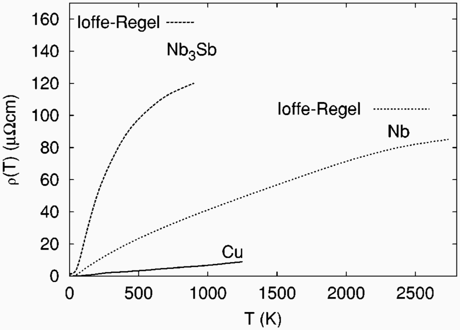
Mott-Ioffe-Regel limit

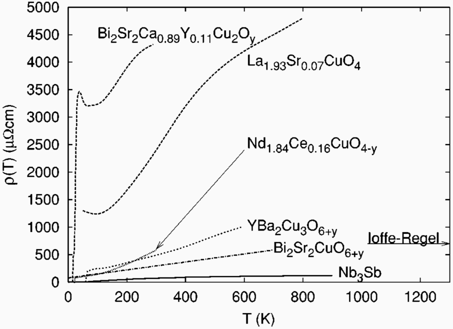
$$\ell = v_F \tau > k_F^{-1} \quad \Rightarrow \quad \tau > \frac{m}{\hbar k_F^2}$$

Saturation of resistivity

$$\rho = \frac{m}{ne^2\tau} < \frac{\hbar k_F^2}{ne^2}$$

Gunnarsson *et al.*, Rev. Mod. Phys. **75**, 1085 (2003)

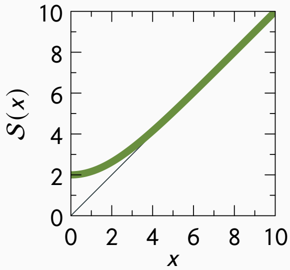


# Self-energy of a strange metal

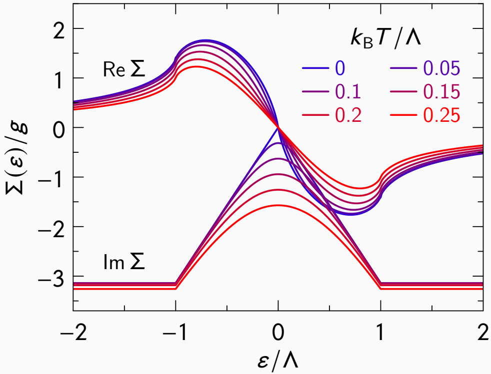
The exactly-solvable models (e.g. SYK) show that the one-particle scattering rate has the scaling form

$$-\mathrm{Im}\,\Sigma(\varepsilon) \propto k_B T\, \mathcal{S}\left(\frac{\varepsilon}{k_B T}\right)$$




The causal (Kramers–Kronig consistent) complex self-energy is

$$\Sigma(z) = g k_B T \int_\Lambda dx\, \frac{\mathcal{S}(x)}{z/k_B T - x}$$

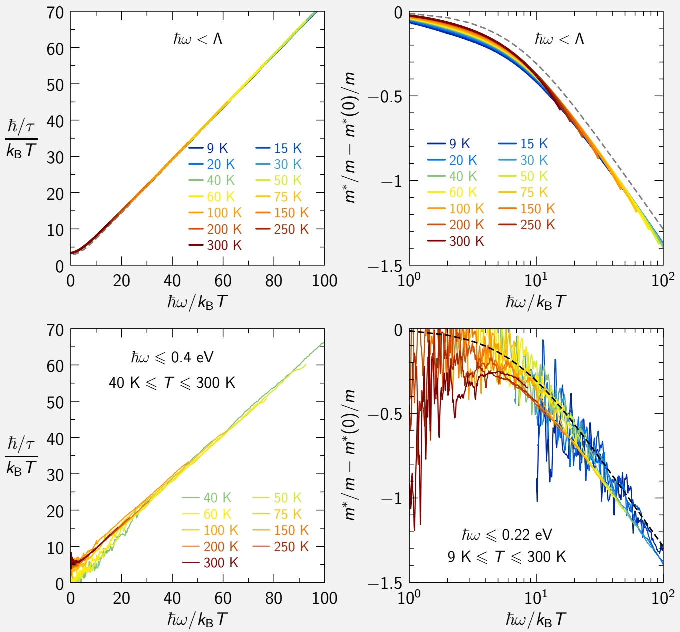


**Planckian Behavior of Cuprate Superconductors: Reconciling the Scaling of Optical Conductivity with Resistivity and Specific Heat**

B. Michon,[1,2,3] C. Berthod,[3] C. W. Rischau,[3] A. Ataei,[4] L. Chen,[4] S. Komiya,[5] S. Ono,[5] L. Taillefer,[4,6] D. van der Marel,[3] and A. Georges[7,8,3,9]

1. Approximate $\omega/T$ scaling in the theoretical conductivity
2. Good $\omega/T$ scaling collapse with similar scaling functions in the data
3. $T$-linear resistivity and $g \log T$ specific heat predicted and observed
4. Power law with anomalous exponent $\nu^*(g) < 1$ predicted in the infrared
5. Anomalous exponent $\nu^*$ in the data consistent with $g$ from the specific heat

**Planckia** … **nductivity**




1. App …
2. Goo …
3. $T$-l …
4. Pow … ared
5. Ano … ific heat

**Planckia... ...nductivity**

1. App...
2. Goo...
3. *T*-l... ...d
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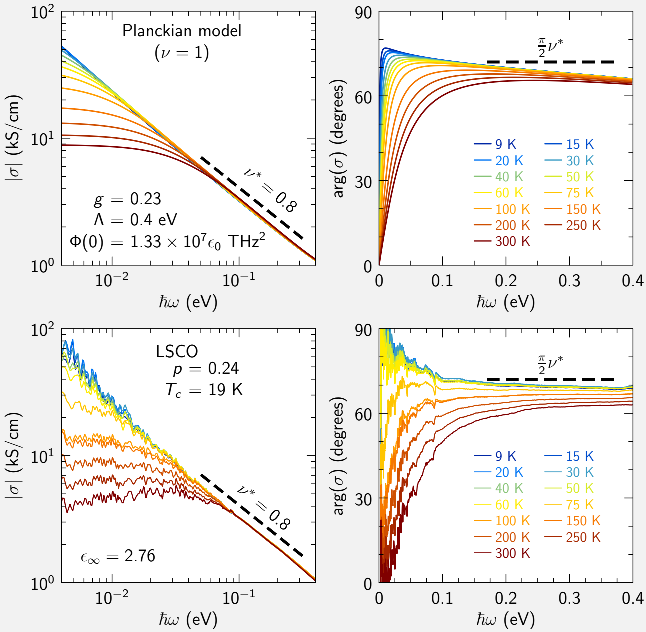
5. Ano... ...ific heat



## [3] arXiv:2205.04030 — Michon *et al.* (Geneva, Sherbrooke)

1. Approximate $\omega/T$ scaling in the theoretical conductivity

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5. Anomalous exponent $\nu^*$ in the data consistent with $g$ from the specific heat

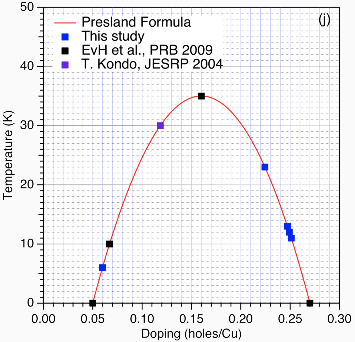
The optical conductivity, resistivity, and specific heat of $La_{2-x}Sr_xCuO_4$, $x = 0.24$, are mutually consistent with a Planckian dissipation scenario.

# [2] PRB 106, 054515 (2022) — van Heumen *et al.* (Amsterdam, Leiden)

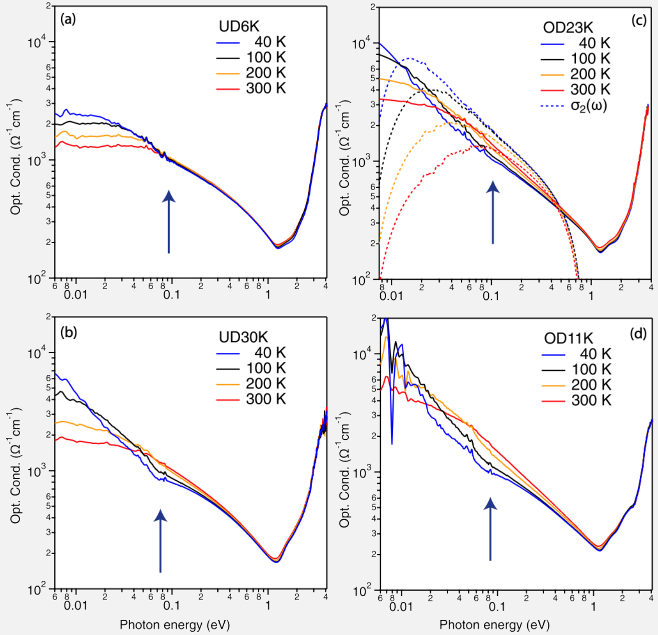
PHYSICAL REVIEW B **106**, 054515 (2022)

**Strange metal electrodynamics across the phase diagram of $Bi_{2-x}Pb_xSr_{2-y}La_yCuO_{6+\delta}$ cuprates**

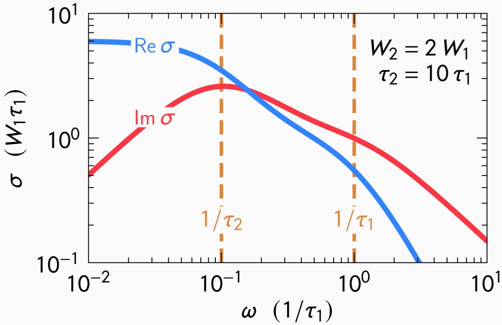
Erik van Heumen,[1,2,*] Xuanbo Feng (馮翾博),[1,2] Silvia Cassanelli,[1] Linda Neubrand,[1] Lennart de Jager,[1] Maarten Berben,[1] Yingkai Huang,[1] Takeshi Kondo,[3] Tsunehiro Takeuchi,[4] and Jan Zaanen[5,†]
[1]*van der Waals - Zeeman Institute, University of Amsterdam, Sciencepark 904, 1098 XH Amsterdam, The Netherlands*
[2]*QuSoft, Science Park 123, 1098 XG Amsterdam, The Netherlands*
[3]*Institute for Solid State Physics, University of Tokyo, Kashiwa-no-ha, Kashiwa, Japan*
[4]*Toyota Technological Institute, Nagoya 468-8511, Japan*
[5]*Lorentz institute, Universiteit Leiden, Leiden, The Netherlands*



# [2] PRB 106, 054515 (2022) — van Heumen *et al.* (Amsterdam, Leiden)



# One-component analysis of a two-component conductivity

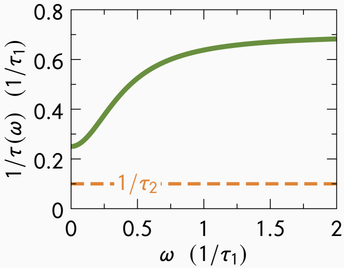


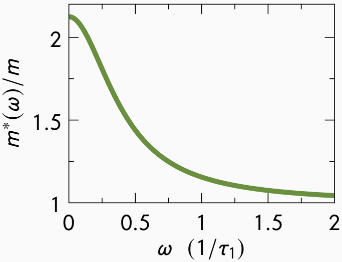

## Two-component Drude model

$$\sigma(\omega) = \frac{W_1}{-i\omega + 1/\tau_1} + \frac{W_2}{-i\omega + 1/\tau_2}$$

## Extended Drude analysis

$$\sigma(\omega) = \frac{W}{-i\omega m^*(\omega)/m + 1/\tau(\omega)}$$



## Two-component analysis

(1) Pure Drude response

(2) "Conformal tail"

Model

$$\hat{\sigma}(\omega) = \hat{\sigma}^D(\omega) + \hat{\sigma}^{\mathrm{inc}}(\omega),$$

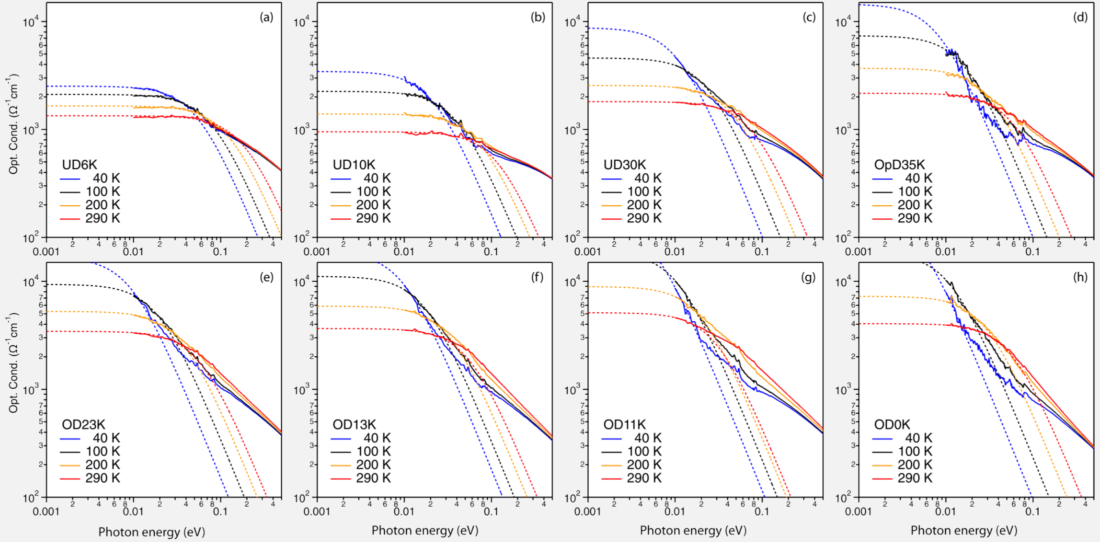
$$\hat{\sigma}^D(\omega) = \frac{D_{\mathrm{Dr}}}{\Gamma_{\mathrm{Dr}} - i\omega},$$

$$\hat{\sigma}^{\mathrm{inc}}(\omega) = \frac{-iD_{\mathrm{inc}}\omega}{(\Delta^2 - \omega^2 - i\Gamma_{\mathrm{inc}}\omega)^\beta}$$

In text books, this Drude response is typically tied to quasiparticles—by reference to the Sommerfeld model—and it was conceptualized like this in this early era. However, such a Drude response is actually completely generic for *any* finite density charged fluid living in a spatial manifold characterized by a weak translational symmetry breaking [20]. It just reflects the fact that the total momentum of the fluid is long-lived (see Sec. II). For instance, the "unparticle" fluids of AdS/CFT

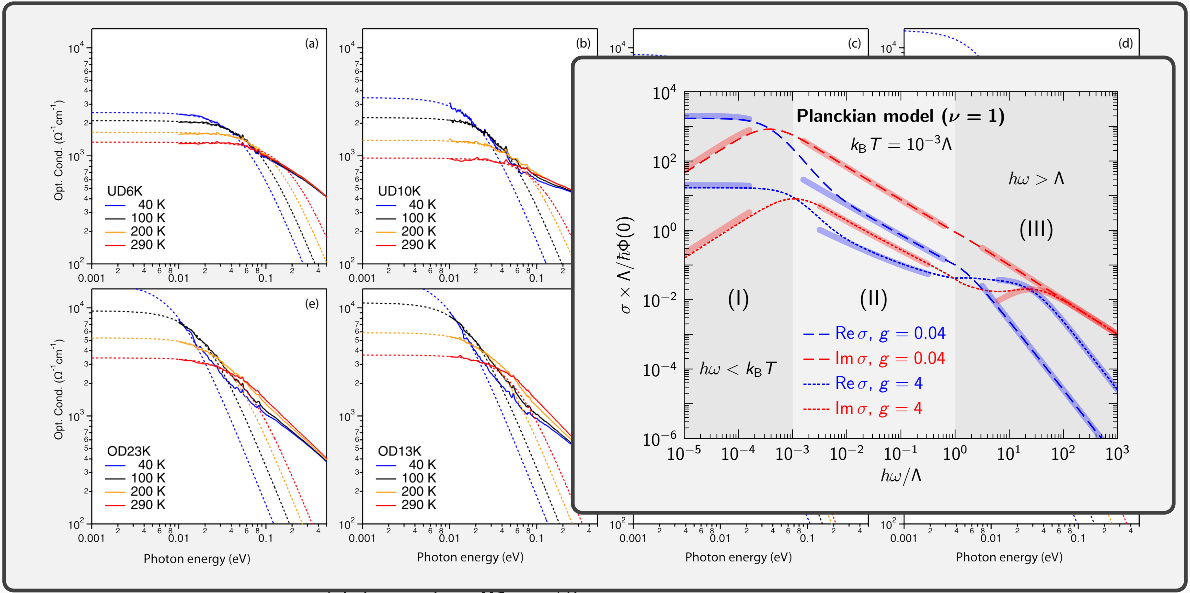
Although the origin of the conformal tail is presently completely in the dark its gross properties may be best understood as reflecting some form of *bound* optical response—it may be viewed as the analog of interband transitions in the strongly interacting electron soup.

# [2] PRB 106, 054515 (2022) — van Heumen *et al.* (Amsterdam, Leiden)




$$(\Delta^2 - \omega^2 - i\Gamma_{\text{inc}}\omega)^\beta$$

# [2] PRB 106, 054515 (2022) — van Heumen *et al.* (Amsterdam, Leiden)




$(\Delta^2 - \omega^2 - i\Gamma_{\mathrm{inc}}\omega)^\beta$

# Optical conductivity of cuprates in a new light

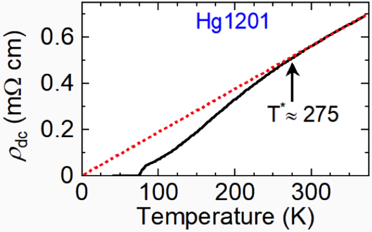
C. M. N. Kumar,[1,2,3] A. Akrap,[4] C. C. Homes,[5,6] E. Martino,[7]
B. Klebel-Knobloch,[1] W. Tabis,[1,3] O. S. Barišić,[2] D. K. Sunko,[8]
N. Barišić[1,8,*]

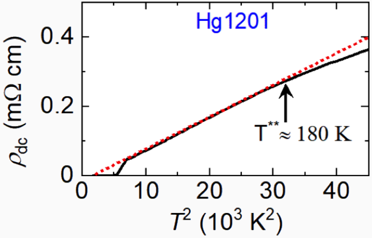
Two-component scenario

(1) Fermi liquid

(2) Localized charges

$$1 + p = n_{\rm eff} + n_{\rm loc}$$



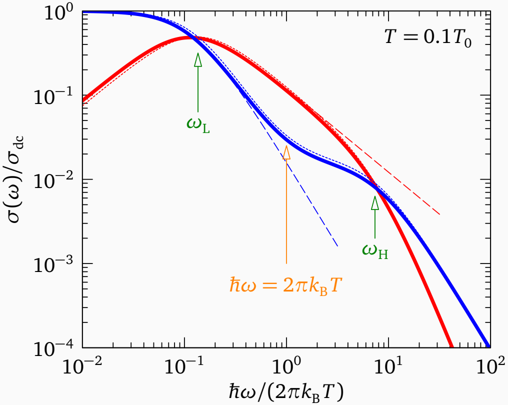
# Optical conductivity of a Fermi liquid

Quasiparticle scattering rate

$$\frac{\hbar}{\tau_{qp}} \propto (\hbar\omega)^2 + (\pi k_B T)^2$$

Optical scattering rate

$$\frac{\hbar}{\tau} \propto (\hbar\omega)^2 + (2\pi k_B T)^2$$

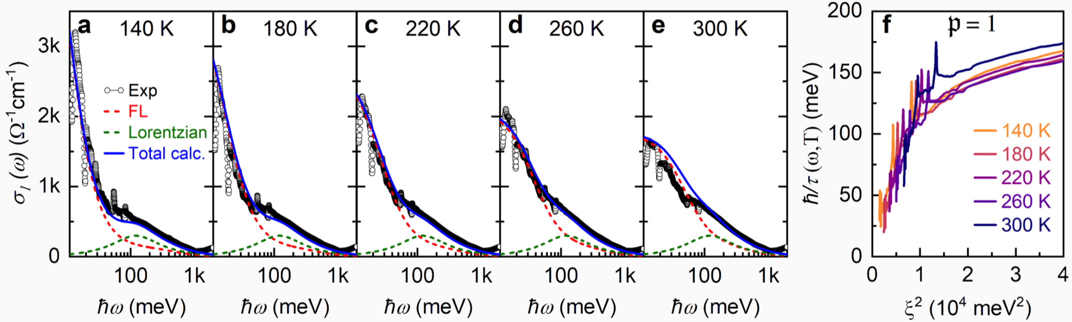



Gurzhi, JETP **35**, 673 (1959)

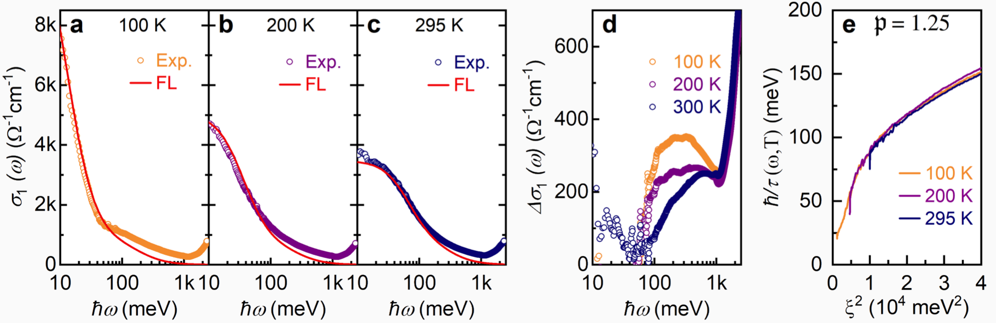
Berthod *et al.*, PRB **87**, 115109 (2013)

## [1] arXiv:2204.10284 — Kumar *et al.* (Vienna, Fribourg, Brookhaven)

Underdoped Hg-1201




Optimally-doped Bi-2212



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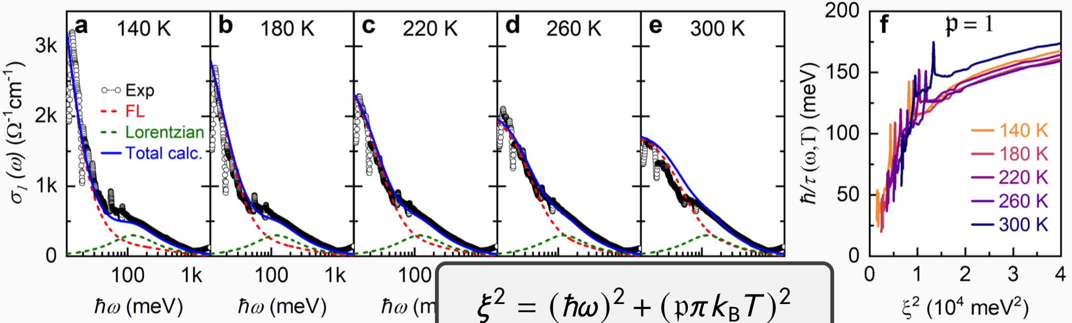
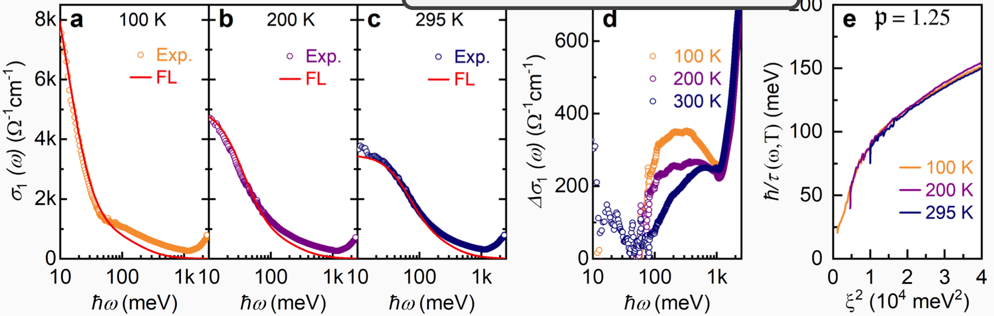
Underdoped Hg-1201



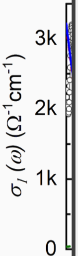

$$\xi^2 = (\hbar\omega)^2 + (\mathfrak{p}\pi k_B T)^2$$

Optimally-doped Bi-2212

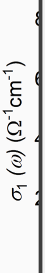


# [1] arXiv:2204.10284 — Kumar *et al.* (Vienna, Fribourg, Brookhaven)

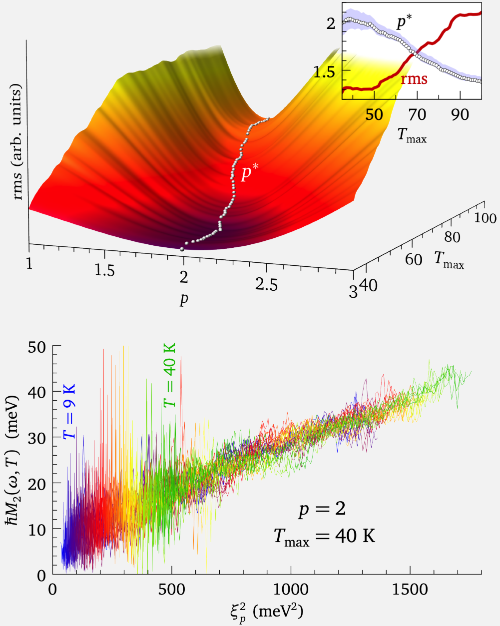
Underdoped Hg-1201



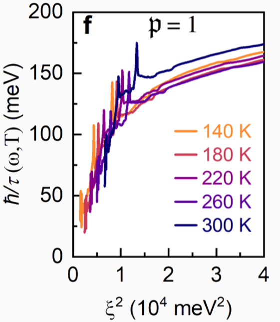

Optimally-doped Bi-2212

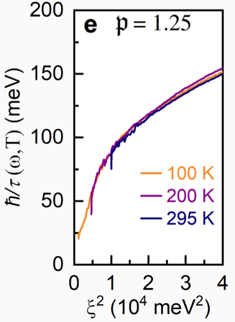



$Sr_2RuO_4$

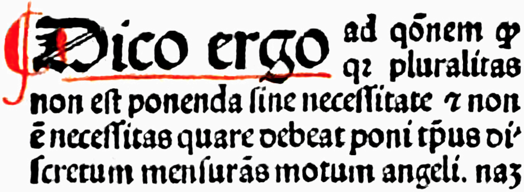



Stricker *et al.*, PRL **113**, 087404 (2014)



*“Pluralitas non est ponenda sine necessitate”*
William of Ockham


Dico ergo ad q̃onem q̄ q2 pluralitas
non eſt ponenda ſine neceſſitate ⁊ non
ē neceſſitas quare debeat poni tp̃us di
ſcretum menſurãs motum angeli. naȝ

Thank you for listening