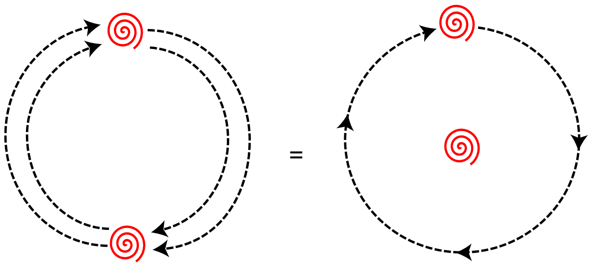
Detection of anyonic braiding statistics in the fractional quantum Hall effect



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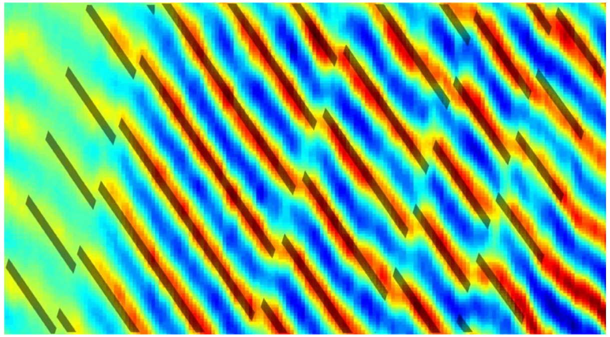


Mainly based on:

Nakamura et al., Nature Physics **16**, 931 (2020)

DQMP Flat Club, 03/12/2021

Hall effect

2D electron gas in a perpendicular magnetic field: $\vec{E} = 0$ $\vec{B} = B\hat{z}$

Go to reference frame that moves with $-\vec{v}$ relative to lab frame

$$\vec{j} = -ne\vec{v} \qquad \vec{E} = -\frac{1}{c}\vec{v} \times \vec{B} \qquad \vec{B} = B\hat{z}$$

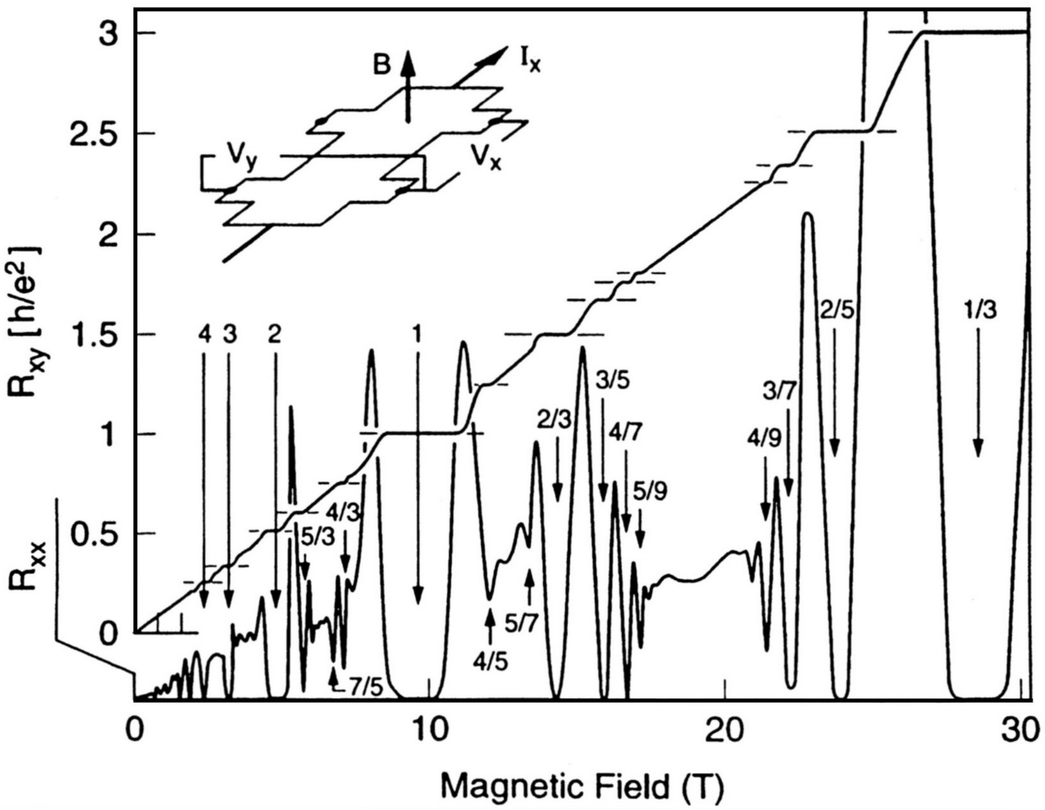
$$\vec{E} = \frac{1}{nec}\vec{j} \times \vec{B}$$

$$\vec{E} = \underline{\underline{\rho}} \cdot \vec{j} \qquad \underline{\underline{\rho}} = \frac{B}{nec}\begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}$$

$$\vec{j} = \underline{\underline{\sigma}} \cdot \vec{E} \qquad \underline{\underline{\sigma}} = \frac{nec}{B}\begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$$

Hall resistivity is linear in the magnetic field, inversely proportional to electron density

The quantum Hall effect – data



$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$\rho_{xy} = \frac{h}{\nu e^2}$$

$$\sigma_{xx} = \rho_{xx} = 0$$

$$\sigma_{xx} \propto e^{-\Delta/T}$$

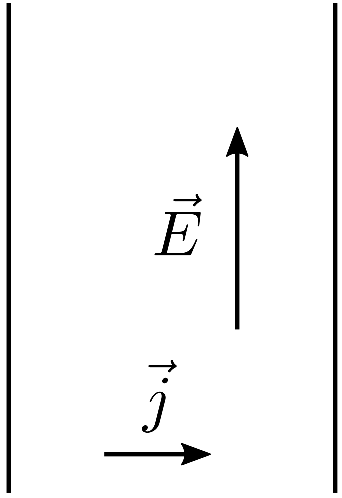
Quantization accuracy:

$$10^{-10}$$

Unavodiable conlcusions

1) Gap must close at the edge

2) Quasiparticles have to have charge νe



$$P \propto \vec{j} \cdot \vec{E}$$

Unavodiable conlcusions

1) Gap must close at the edge

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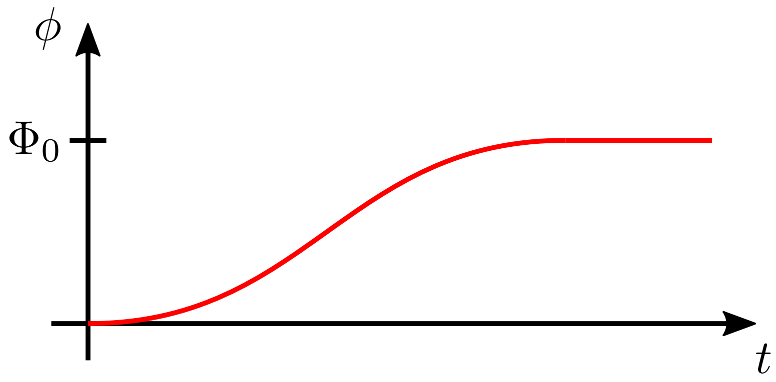
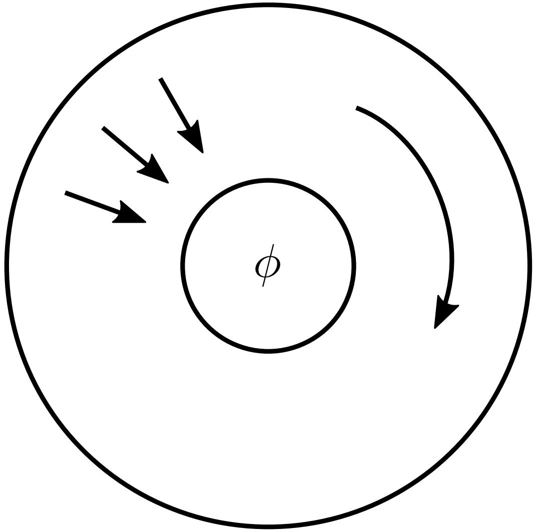




$$\oint \vec{E} d\vec{l} = -\frac{d\phi}{dt} \quad \rightarrow \quad \vec{E} = -\frac{1}{2\pi r}\frac{d\phi}{dt}\hat{\theta}$$

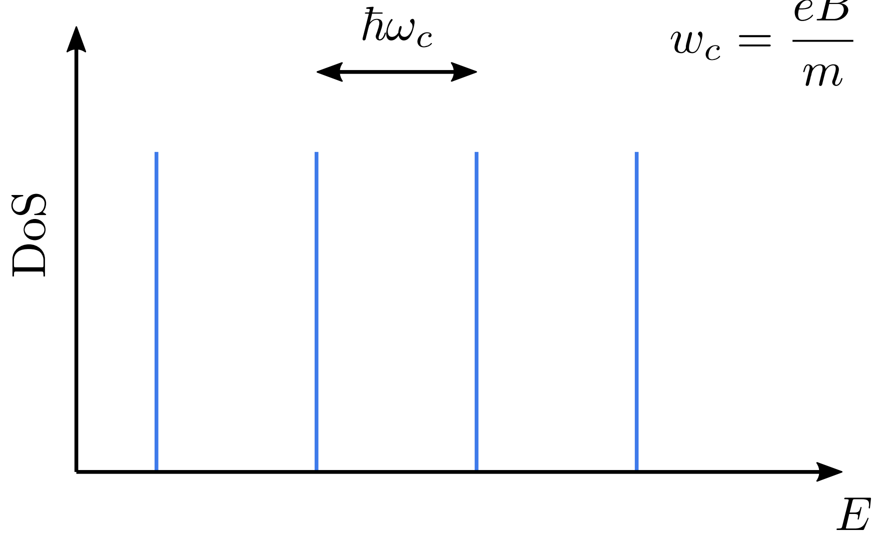
$$\vec{j} = \underline{\underline{\sigma}} \cdot \vec{E} = -\nu \frac{e^2}{h}\frac{1}{2\pi r}\frac{d\phi}{dt}\hat{r}$$

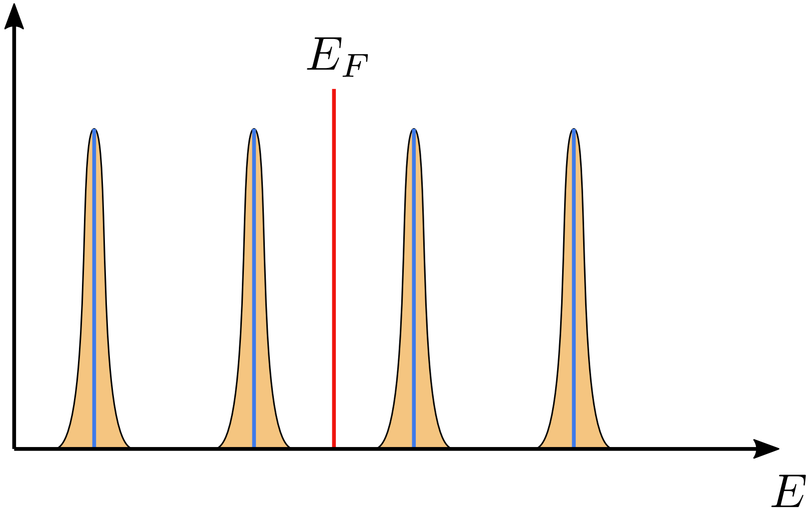
$$\Delta Q = \int_0^t dt' \int_0^{2\pi} r d\theta j = -\nu \frac{e^2}{h}\Phi_0 = \boxed{-\nu e}$$

Landau levels

Quantum mechanical treatment of particles in magnetic field in 2D

$$w_c = \frac{eB}{m}$$





with disorder and finite T

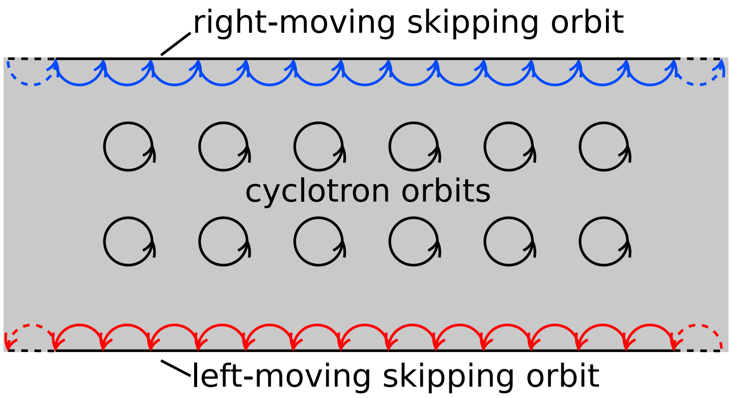
Degeneracy: $N_\phi = \frac{\phi}{\Phi_0} = \frac{BA}{\Phi_0}$

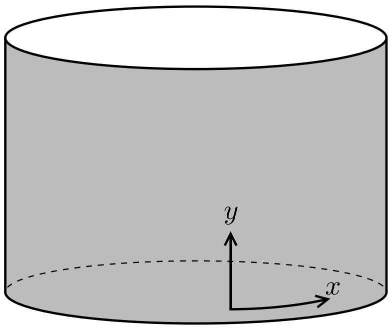
$$\Phi_0 = \frac{h}{e}$$

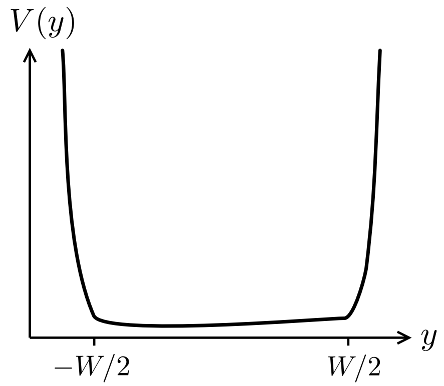
Filling factor $\nu : N_e/N_\phi = n_e/B$

Edge states

Semiclassical picture: skipping orbits

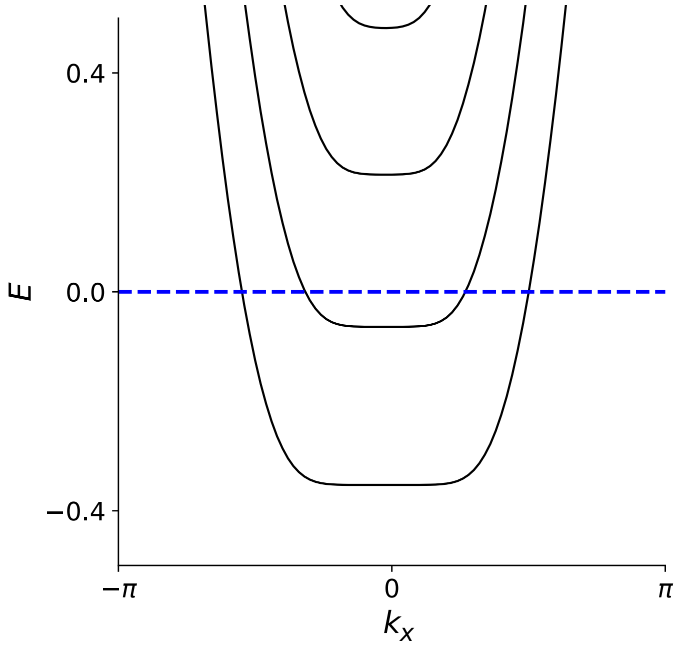






Landau level picture

$$\psi_{n,k_x} \propto e^{ik_x x} H_n(y + k_x l_B^2) e^{-\frac{1}{2l_B^2}(y+k_x l_B^2)^2}$$



Fractional quantum Hall effect (FQHE)

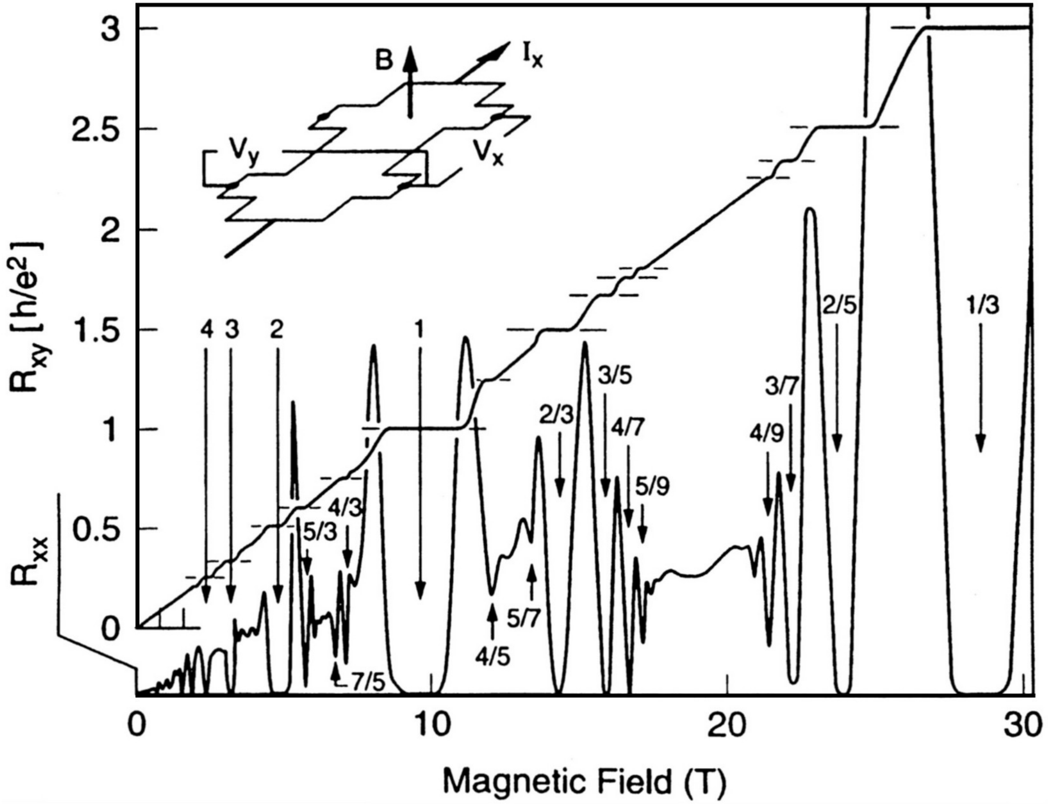
Phenomenology very similar

- Quantization of σ_{xy}
- Chiral edge states

But...

FQHE is a genuine many-body state

Fundamentally different from integer QHE



Statistics of quasiparticles

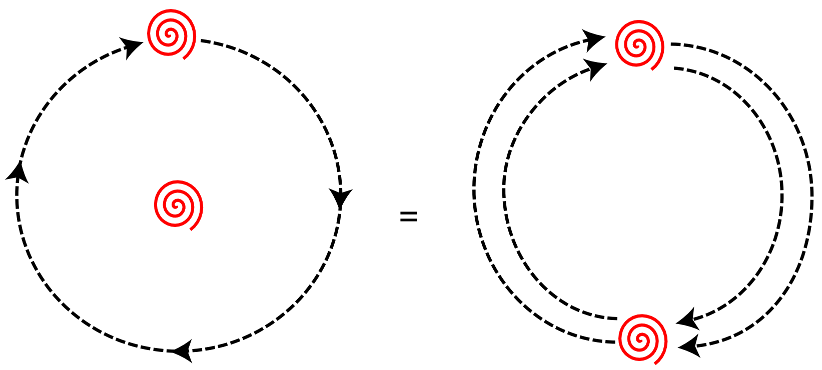
Laughlin wave function for filling factor $1/m$

Laughlin 1983

$$\psi_m = \prod_{j<k} (z_j - z_k)^m e^{-\frac{1}{4}\sum_l |z_l|^2}$$

With a quasihole at z_0

$$\psi_m^{+z_0} = N_+ \prod_i (z_i - z_0)\psi_m$$



Adiabatically moving one quasihole around another leads to phase

$$\Delta\gamma = 2\pi\nu$$

Halperin 1984; Avoras, Schrieffer and Wilczek 1984

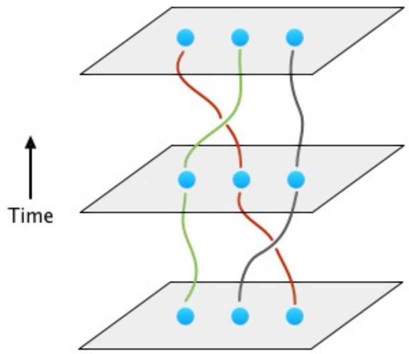
General: Topological order

- Long-range entangled quantum state in (2+1)D characterized by
  - (1) a chiral central charge, and
  - (2) a unitary modular tensor category (UMTC)

- UMTC contains topological charges (quasiparticles) and their fusion and braiding rules
  → quasiparticles can be non-Abelian

Topological order

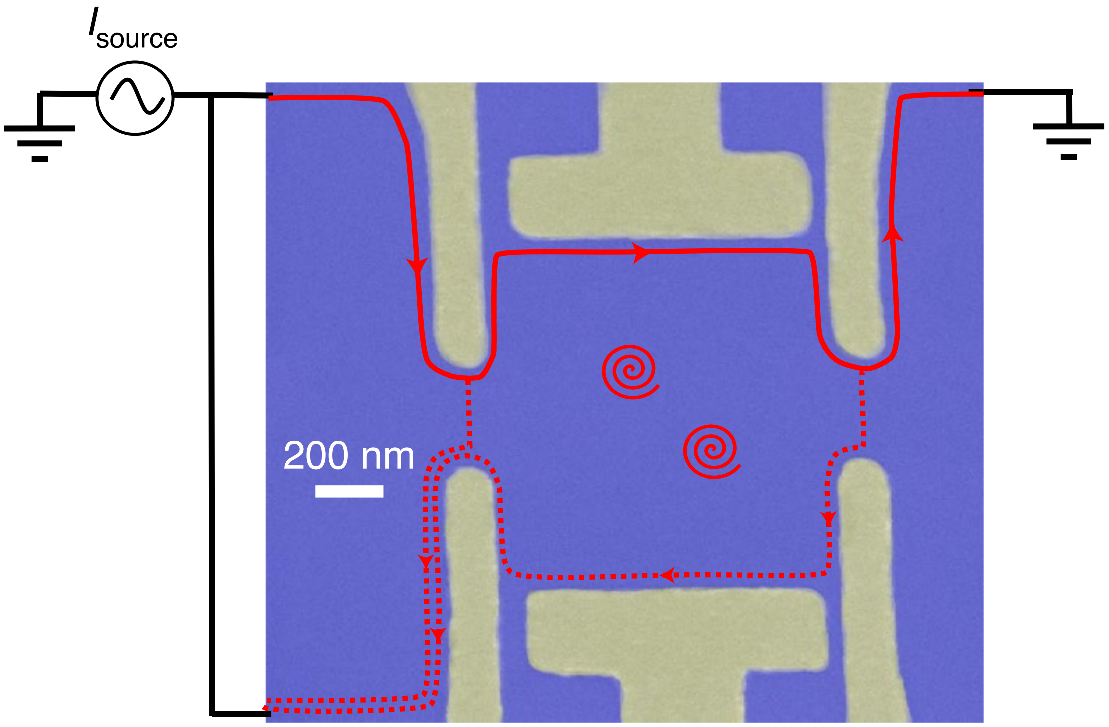
- Fractional quantum Hall effect
- Some spin liquids
  (Chiral spin liquids, Kitaev's toric code, ...)



No topological order

- Topological insulators
- Majorana wires
- ...

Quantum Hall Fabry-Perot interferometer



Phase:

$$\theta = 2\pi \frac{e^*}{e} \frac{A_I B}{\Phi_0} + N_{\text{qp}} \theta_{\text{anyon}}$$

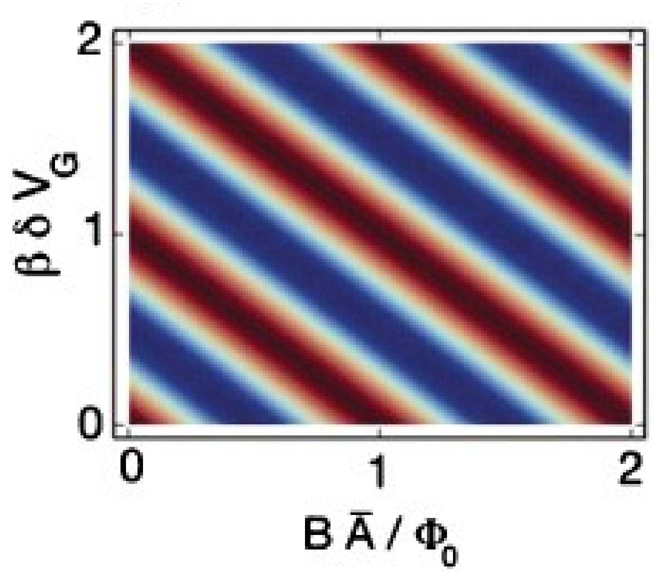
Conductivity:

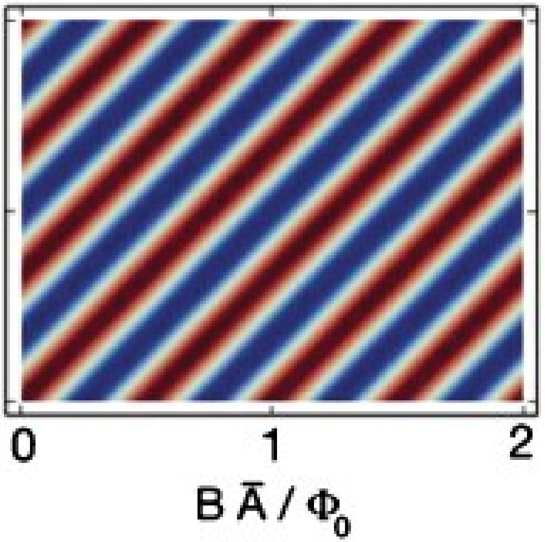
$$G \propto \cos(\theta)$$

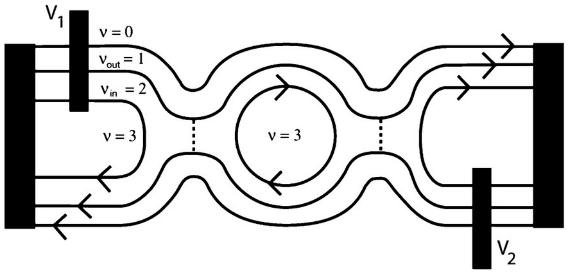
Different regimes

Aharonov-Bohm regime

Coulomb-dominated regime







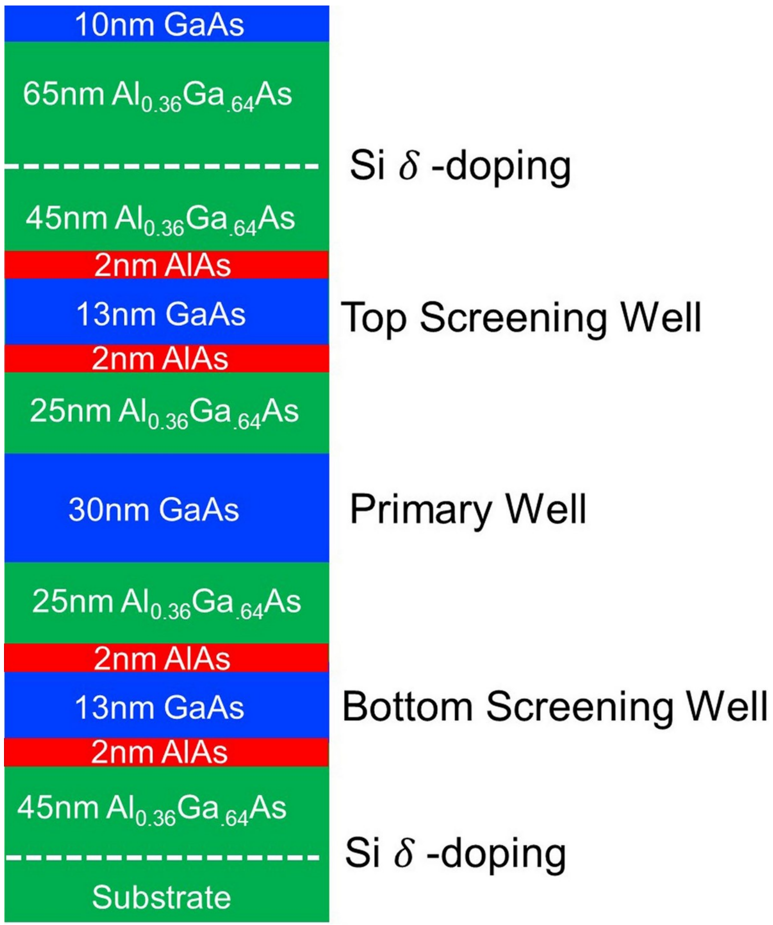
$$\theta = 2\pi \frac{e^*}{e} \frac{A_I B}{\Phi_0}$$

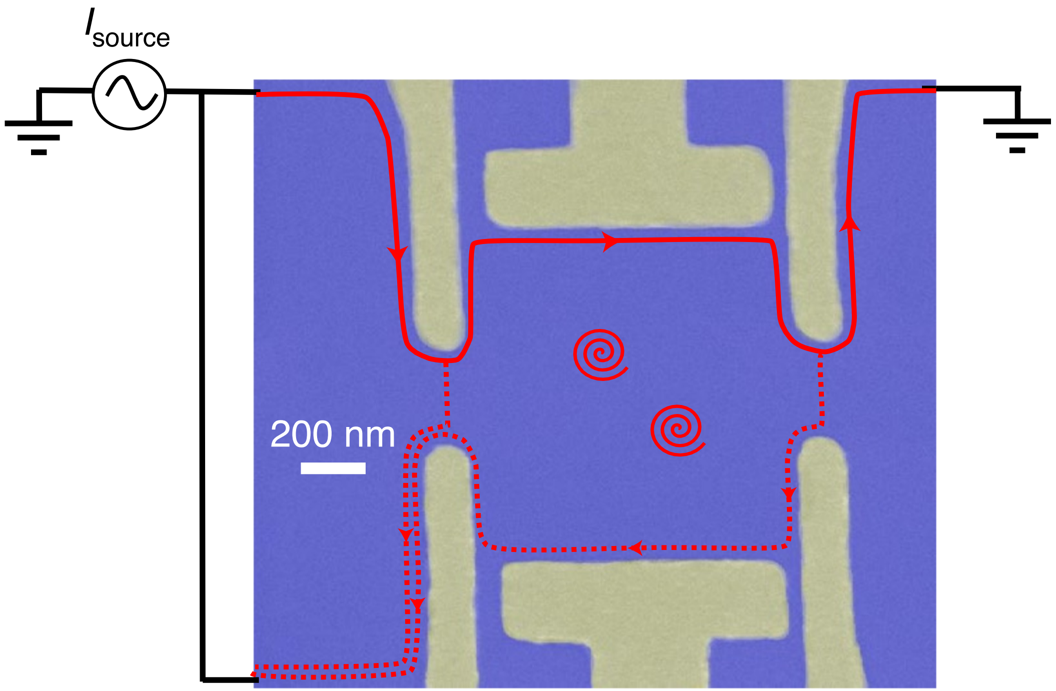
$$A_I = \overline{A}(B, V_G) + \delta A_I$$

$$\phi = B\overline{A}/\Phi_0$$

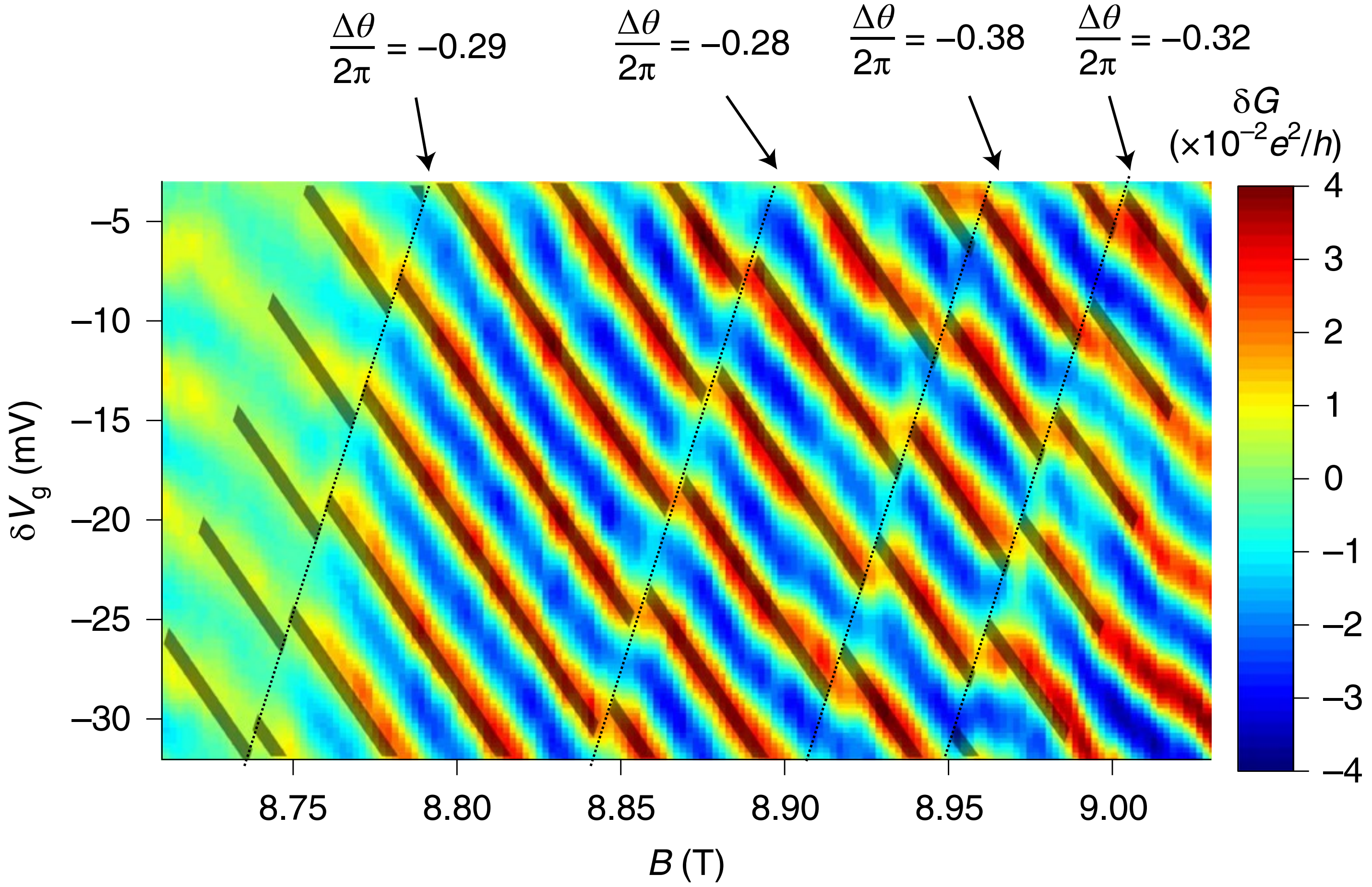
$$\delta R = \mathrm{Re}\left(\sum_{-\infty}^{\infty} R_m e^{2\pi i (m\phi + \alpha_m \delta V_G)} \right)$$

Experimental setup





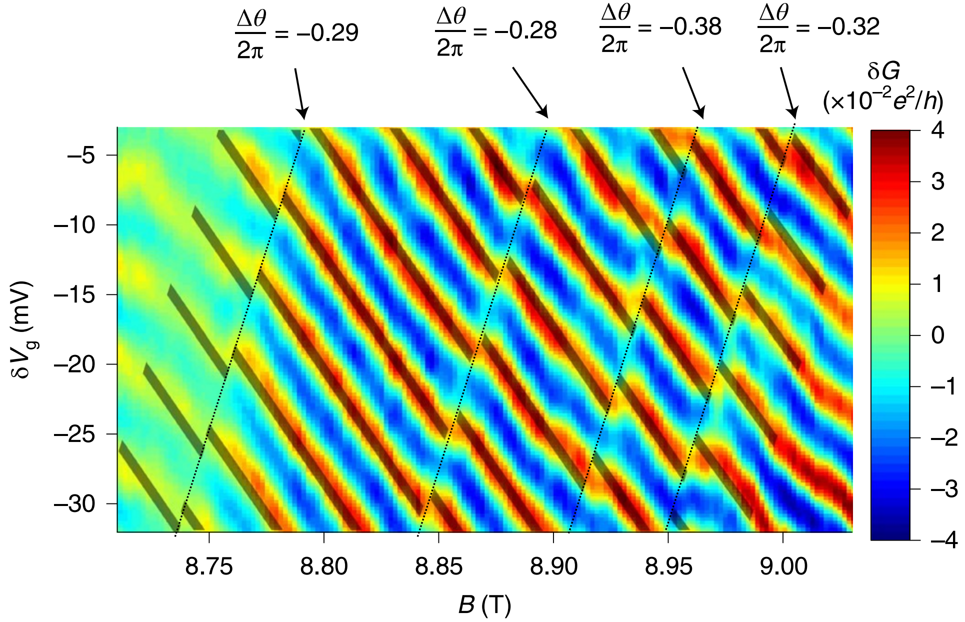
Anyon signatures



Anyon signatures

- Lines of constant phase with negative slope → AB regime

- Discrete phase slips
  → quasihole/-particle creation/removal
  → random distance in B



$$\delta G = \delta G_0 \cos\left(2\pi \frac{1}{3}\frac{A_I B}{\Phi_0} + \theta_0\right)$$

$$\theta_{\text{anyon}} = 2\pi \times (0.31 \pm 0.04)$$

- Jumps across lines with positive slope
  → Increase in B removes quasiparticles
  → Increase in V_g favors localized quasiparticles

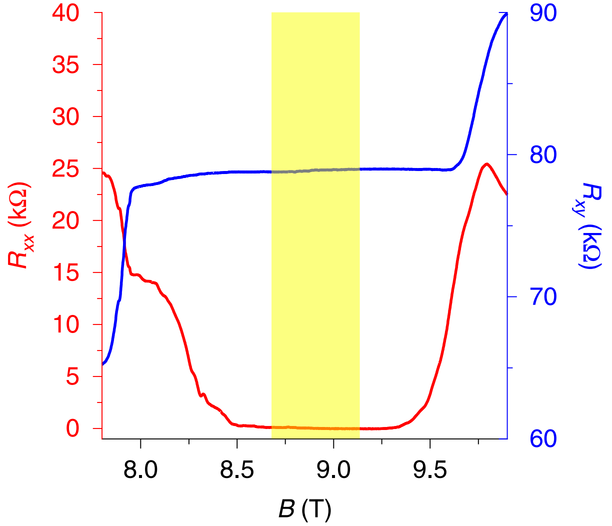
Rosenow-Stern theory

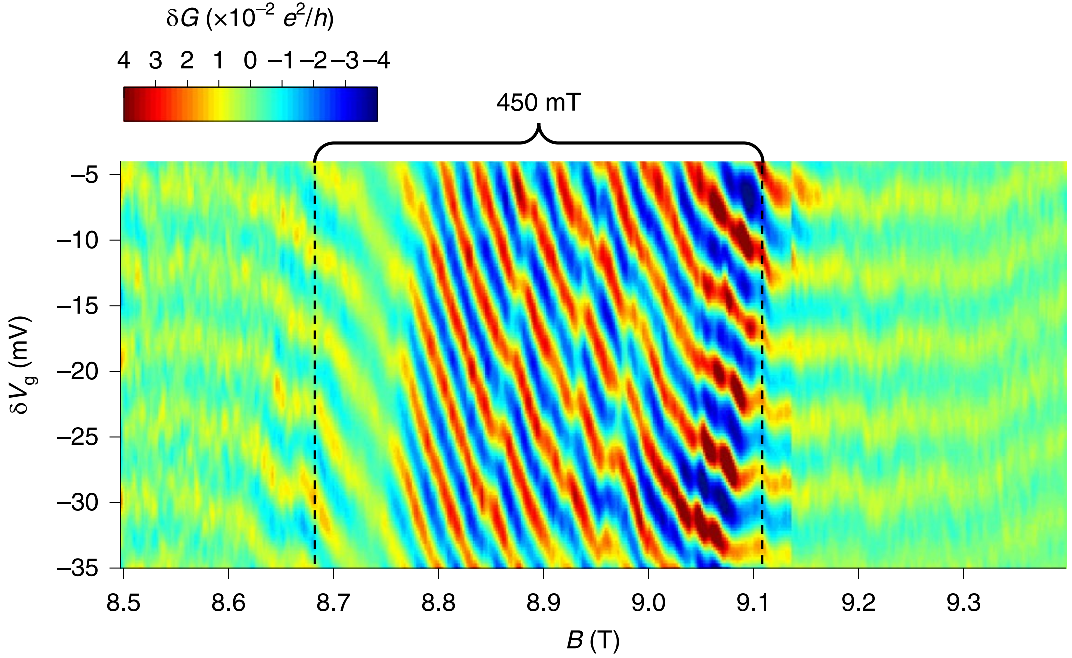
Rosenow and Stern 2020

- Theory for interferometer in $\nu = 1/3$ state with strong screening
- Predicts two regimes as function of magnetic field

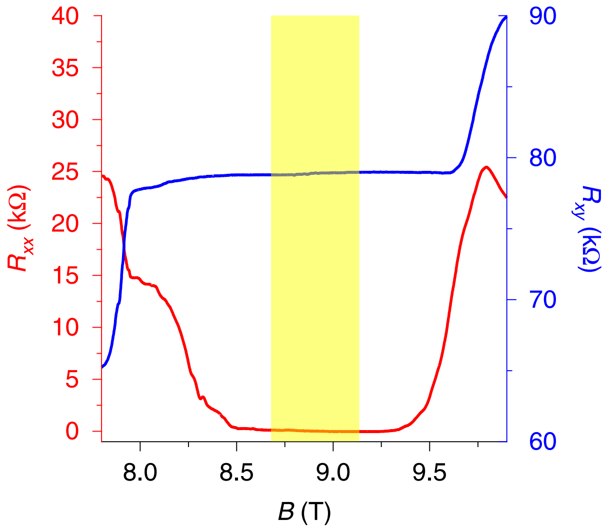
  → center of plateau: constant filling

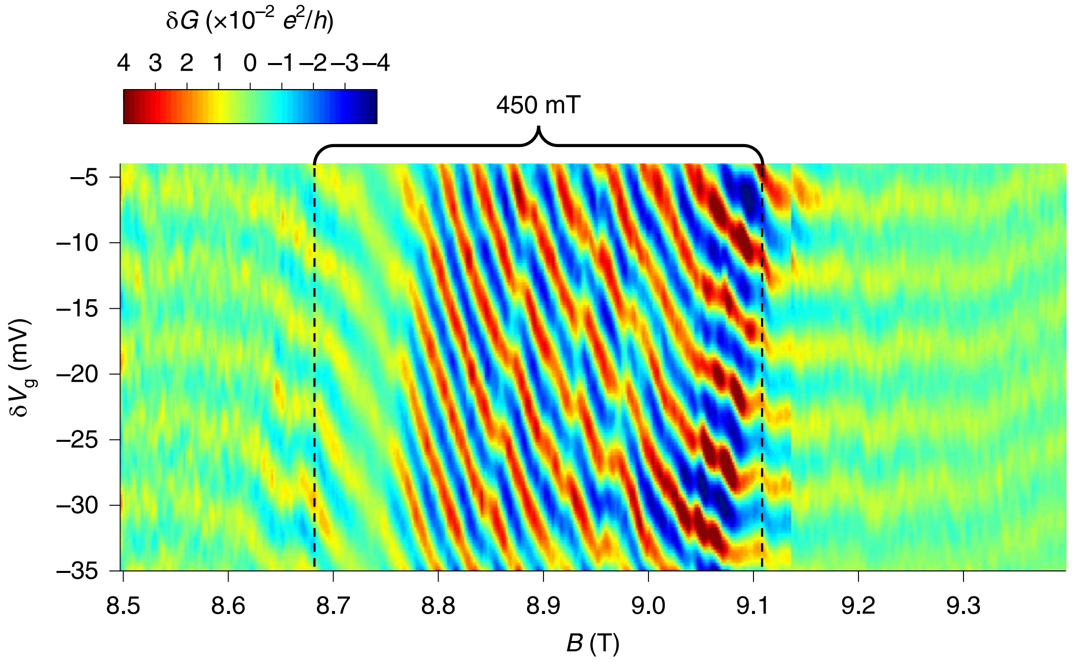
  → edges of plateau: constant density – many quasiparticles/-holes created/removed (1 per change of flux Φ_0)





Rosenow-Stern theory





Why barely dependent on B?

- Adding one flux quantum leads to additional Aharonov-Bohm phase of $2\pi/3$
- Simultaneously one quasiparticle (quasihole) will be removed (added) in the low (high) field regime → phase shift of $-2\pi/3$
- Thermal smearing makes it continuous

Rosenow-Stern theory

Width of constant filling factor estimate

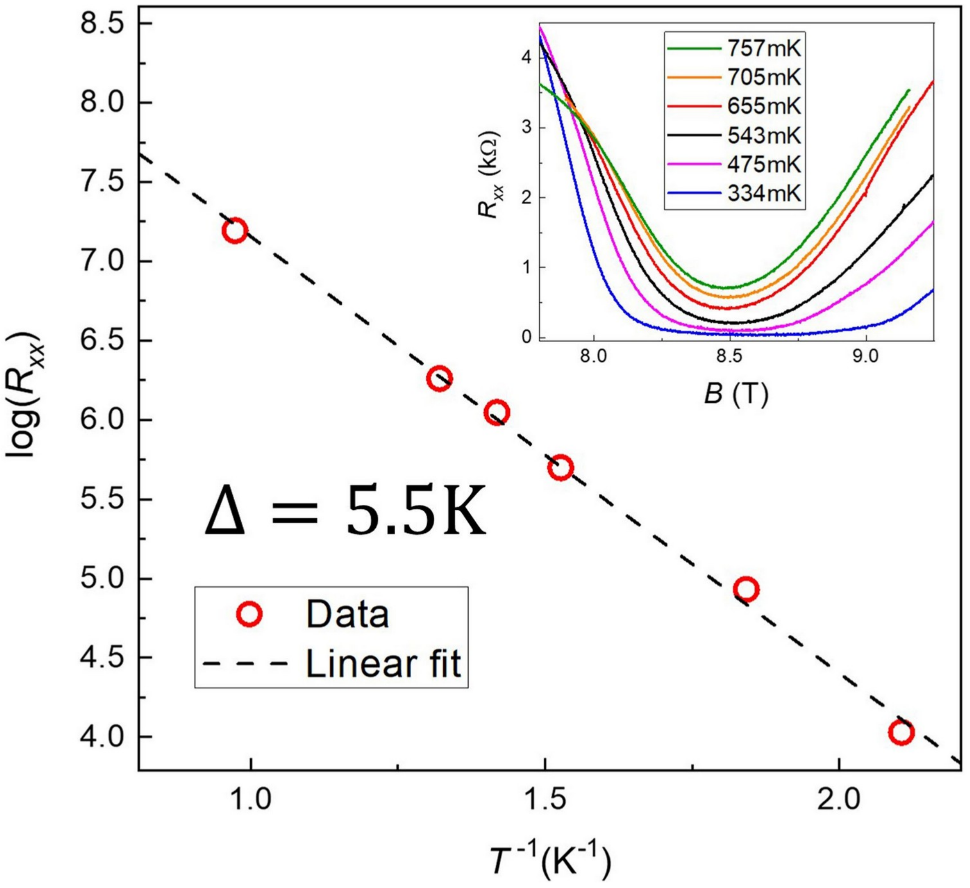
$$\Delta B_{\text{constant}\,\nu} = \frac{\Delta_{1/3}\Phi_0 C_{\text{SW}}}{\nu e^2 e^*}$$

$\Delta_{1/3}$ – gap

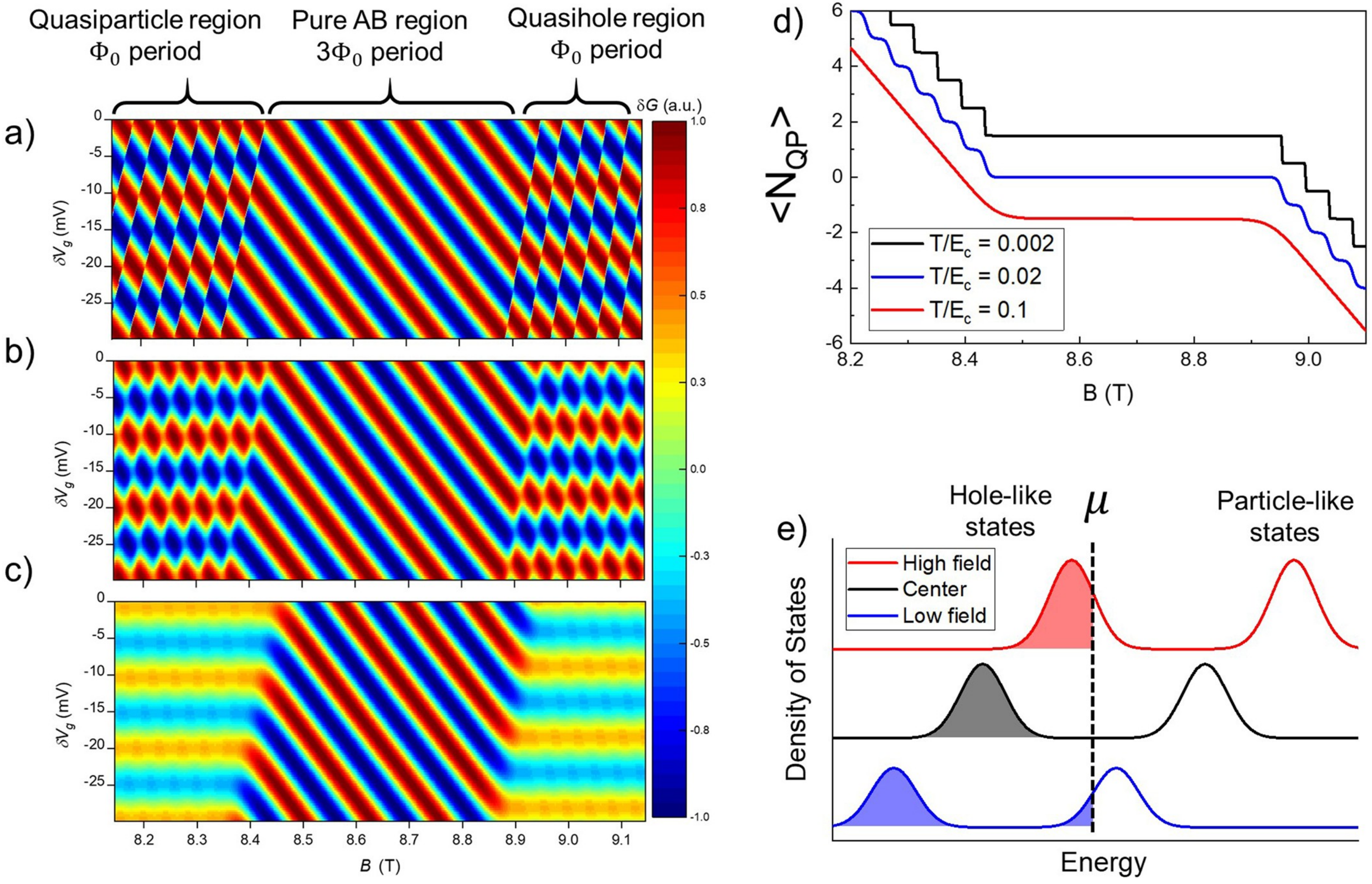
$C_{\text{SW}} = \frac{2\epsilon}{d}$ – Capacitance per unit area of the screening layers

Theory: ≈ 530 mT

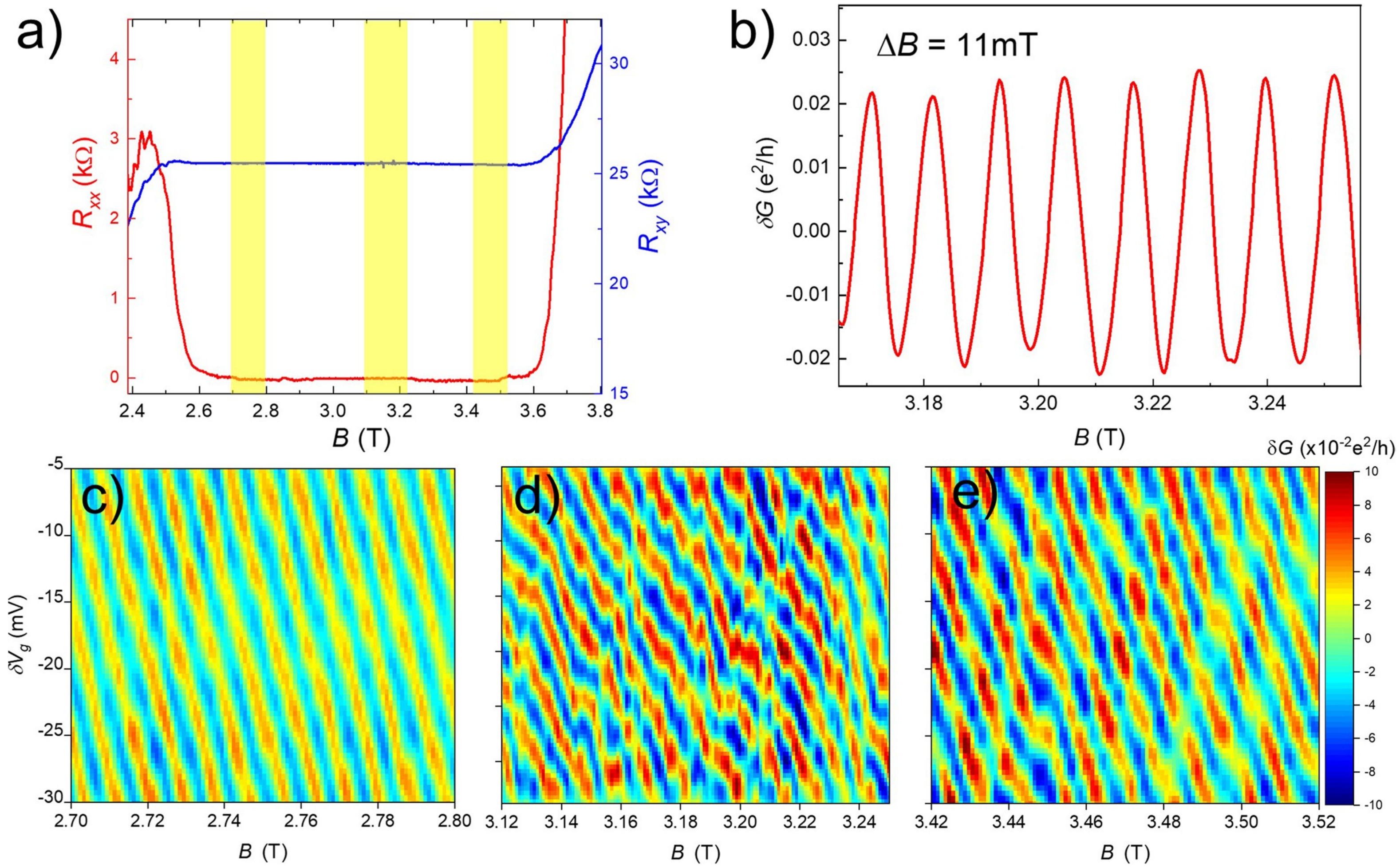
Experiment: ~ 450 mT



Simulations



Comparison to $\nu = 1$

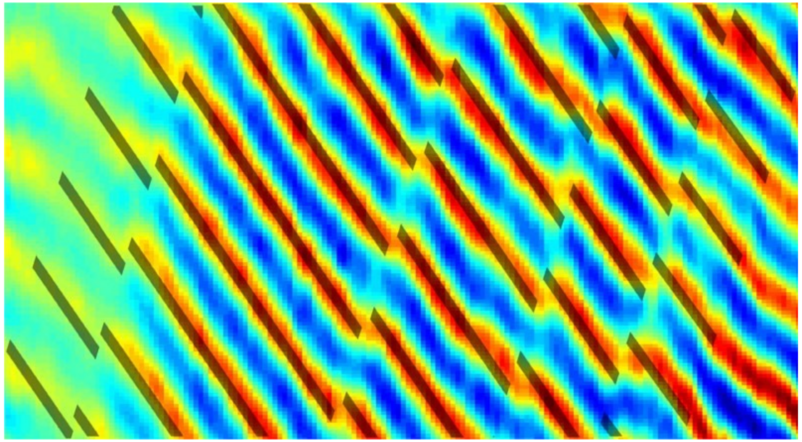


Summary: Anyon braiding in the FQH

- Quantum Hall Fabry-Perot interferometer in Aharonov-Bohm limit

- Discrete phase slips provide signal for anyonic quasiparticles

- Different behavior at weaker/stronger field compared to central plateau consistent with theory



Open question

- Non-Abelian braiding?