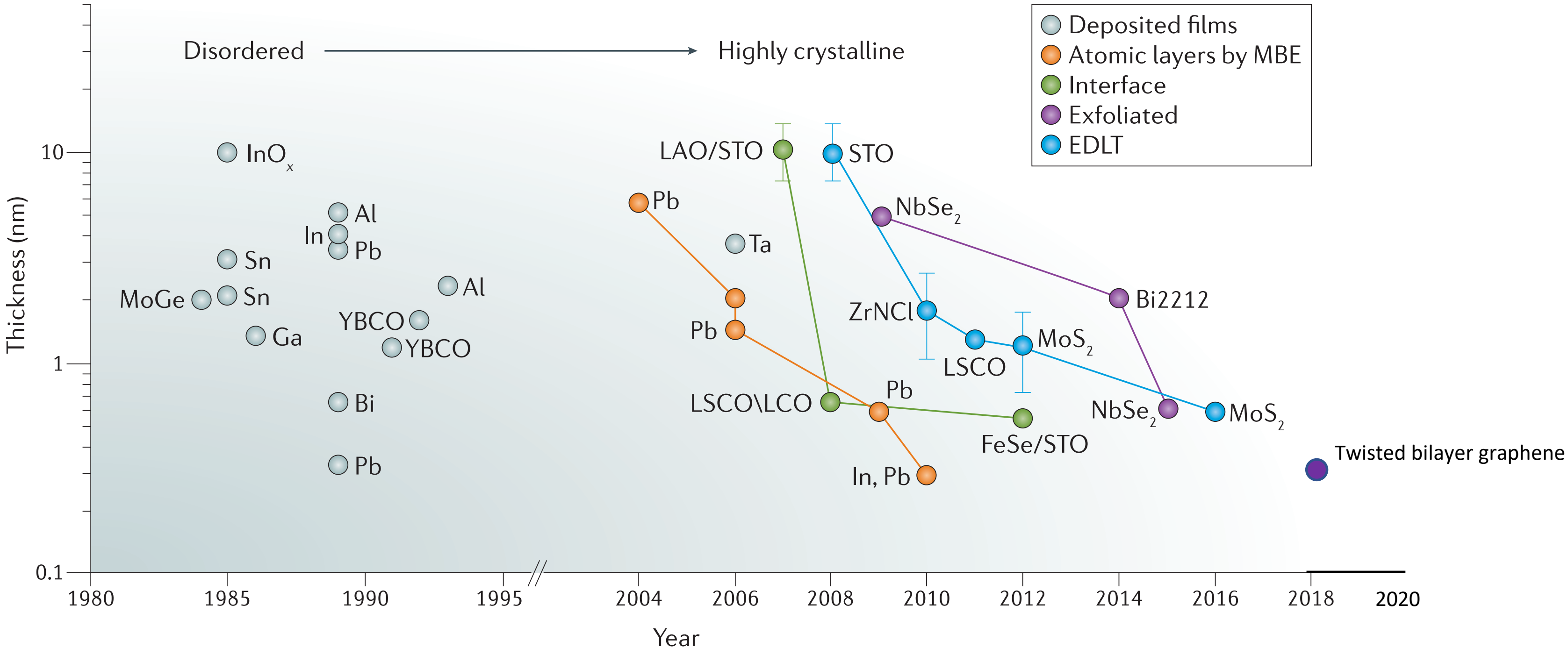
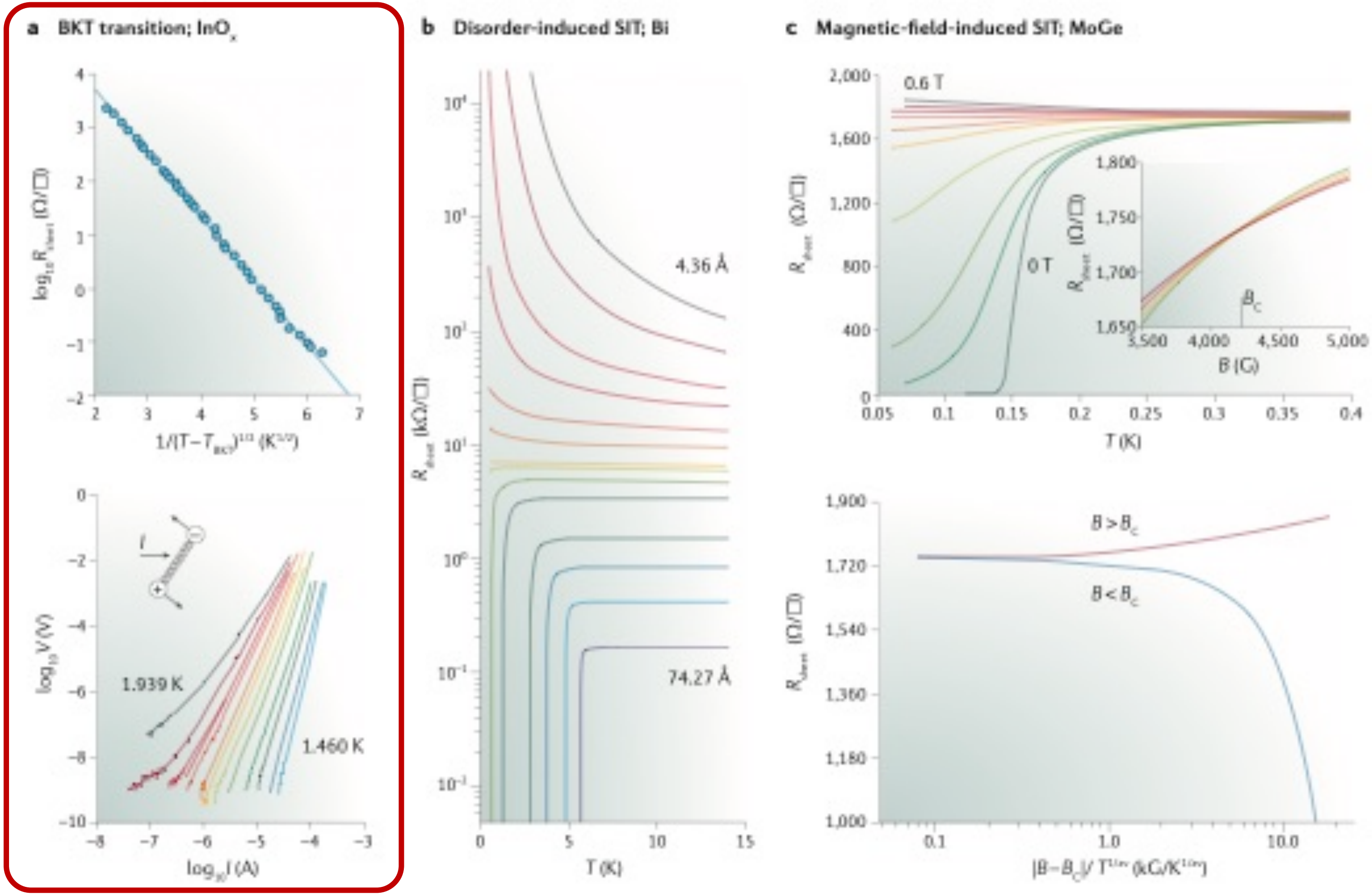
2D superconductivity

22 October 2021

2D Superconductivity: History





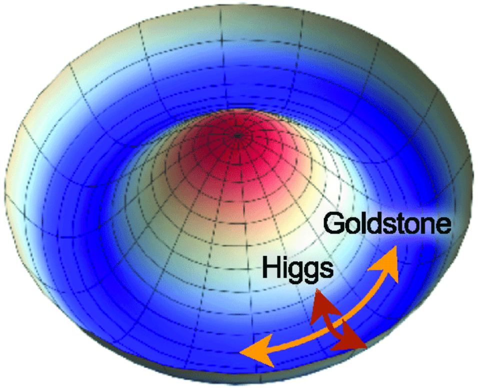
a BKT transition; InOx
log10 Rsheet (Ω/□)
1/(T−TBKT)^1/2 (K^−1/2)
log10 V (V)
log10 I (A)
1.939 K
1.460 K
b Disorder-induced SIT; Bi
Rsheet (kΩ/□)
T (K)
4.36 Å
74.27 Å
c Magnetic-field-induced SIT; MoGe
0.6 T
0 T
Rsheet (Ω/□)
T (K)
B (G)
Bc
B > Bc
B < Bc
|B−Bc|/ T^1/zν (kG/K^1/zν)

Ginzburg-Landau theory of superconductivity

Cooper pair $\Psi(x) = \langle c_{x\uparrow} c_{x\downarrow} \rangle = |\Psi(x)| e^{i\varphi(x)}$

Free energy functional $\mathcal{F} = \int d^2x \left[\frac{1}{2} |\nabla \Psi(x)|^2 + \frac{1}{2} r(T) |\Psi(x)|^2 + \frac{1}{4} u |\Psi(x)|^4 \right]$

At low temperatures a **Mexican hat potential** when *r(T) < 0* and *u > 0*



Superfluid density $\rho_s \equiv |\Psi|^2 = |r(T)|/u$

Goldstone mode from phase variation

$$\mathcal{F} = \frac{1}{2} \rho_s \int d^2x \, (\nabla \varphi(x))^2$$

Mermin-Wagner theorem

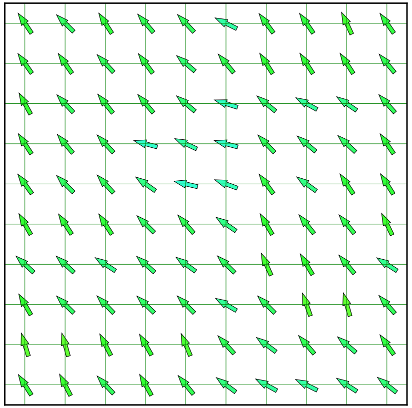
Fluctuations of order parameter $\langle \delta\Psi(x)\delta\Psi(y)\rangle$

Determined by Goldstone modes $G(k, i\omega_n) = \dfrac{1}{\omega_n^2 + k^2}$

Diverges at finite temperature $(\delta\Psi)^2 \sim T \sum_n \int d^d k \; G(k, i\omega_n) \sim T \int dk \; k^{d-3}$

No long-range order in two dimensions for T>0

XY model and algebraic long-range order

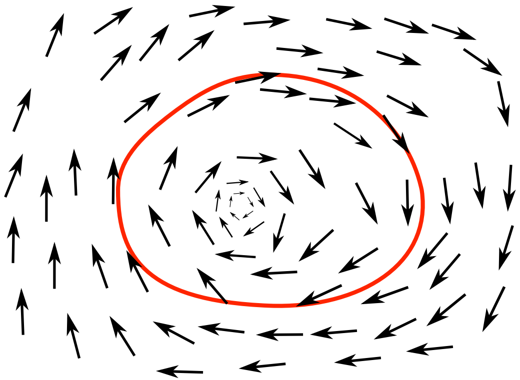


XY model: phase of the order parameter is a in-plane vector

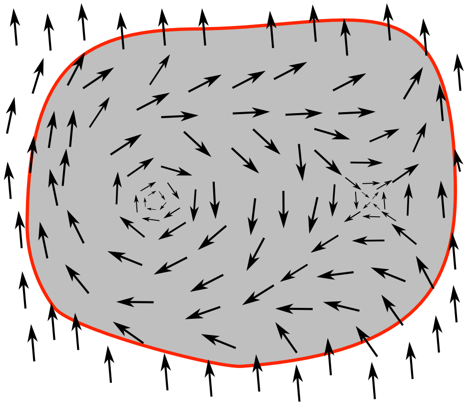
At low temperature there is **algebraic long-range order**

$$G(r) = \langle e^{-i\varphi(r)} e^{i\varphi(0)} \rangle \sim r^{-\eta(T)}$$

with $\eta(T) = \dfrac{k_B T}{2\pi\rho_s}$



But you can also have
vortices
and
vortex-antivortex pairs



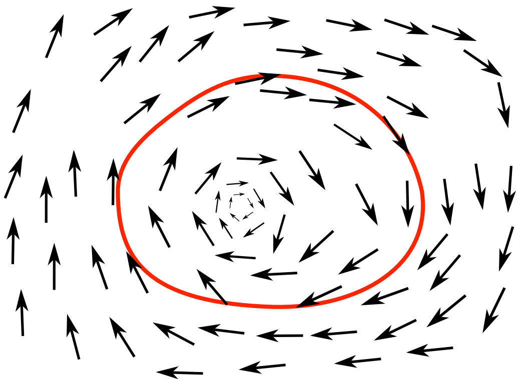
Ref: Goldman 2013

BKT transition

A vortex satisfies $|\nabla\varphi| = 1/r$

The **energy** of a single vortex is $E = \frac{1}{2}\rho_s \int d^2x\, (\nabla\varphi(x))^2 = \pi\rho_s \int_a^L \frac{dr}{r} = \pi\rho_s \log\frac{L}{a}$

And the **entropy** of a vortex is $S = \log\frac{L^2}{a^2}$



The **free energy** of a vortex is thus

$$F = E - ST = (\pi\rho_s - 2k_BT)\log\frac{L}{a}$$

Therefore you can have free vortices proliferate at

$$T_{BKT} = \frac{\pi}{2}\rho_s$$

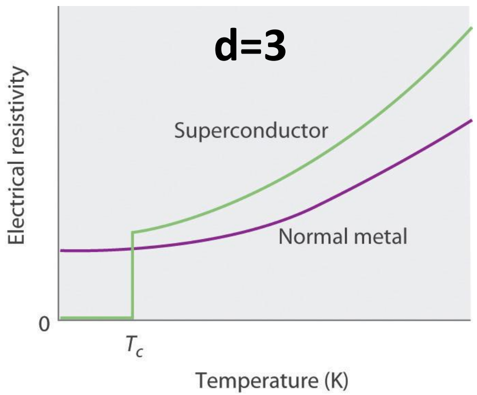
Superconductivity and resistivity

Supercurrent is given by **phase gradient** $J = \frac{2e}{\hbar}\rho_s \nabla\varphi$

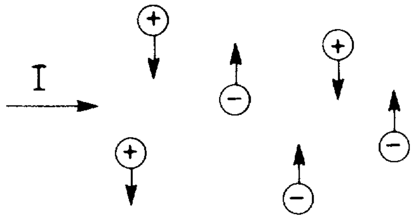
A supercurrent causes a sideways **force** on a vortex $\vec{F} = n_W \frac{h}{2e}\vec{J} \times \hat{z}$

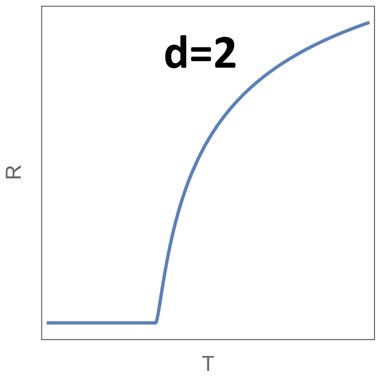
Moving free vortices cause **resistivity** $\rho = \left(\frac{h}{2e}\right)^2 n_v \mu_v$

Halperin-Nelson formula : resistivity above T_{BKT} is proportional to **density of vortices**

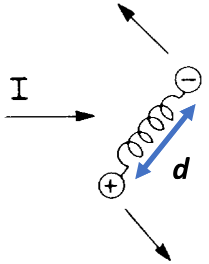


$$R = R_0 \exp\left(-b|T - T_{BKT}|^{-1/2}\right)$$





I/V curve below T_{BKT}



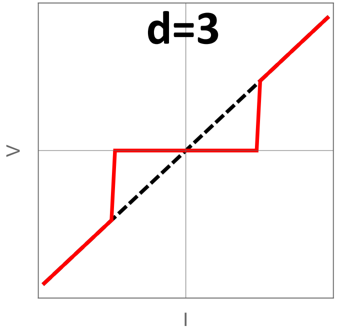
In 2d, below TBKT, vortex **pairs** are bound with energy $U = 2\pi\rho_s \log\frac{d}{a}$

An applied **current** tries to pull them apart $U = -\frac{h}{2e}Jd$

There is a **maximum** energy at *d_0* above which vortex-pairs can **break**!

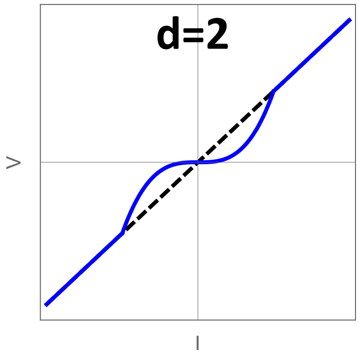
$$U(d_0) = 2\pi\rho_s \log\frac{J_0}{J}$$

In equilibrium, the **vortex density** is given by rate $n_v \sim e^{-U(d_0)/2T} \sim J^{1/2\eta(T)}$

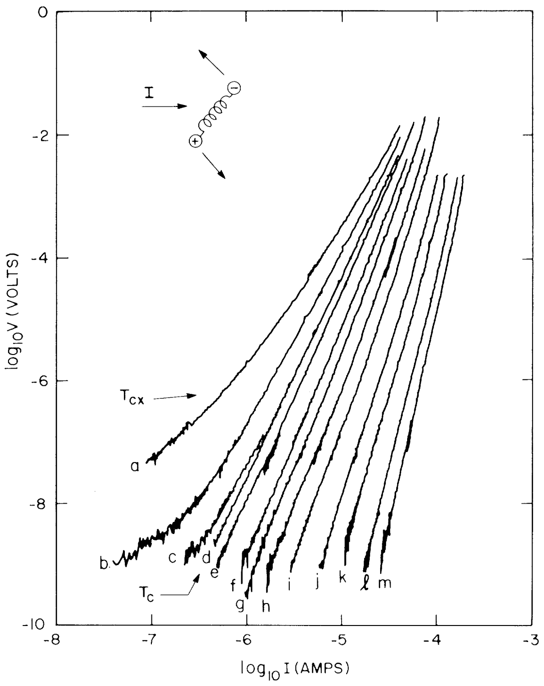


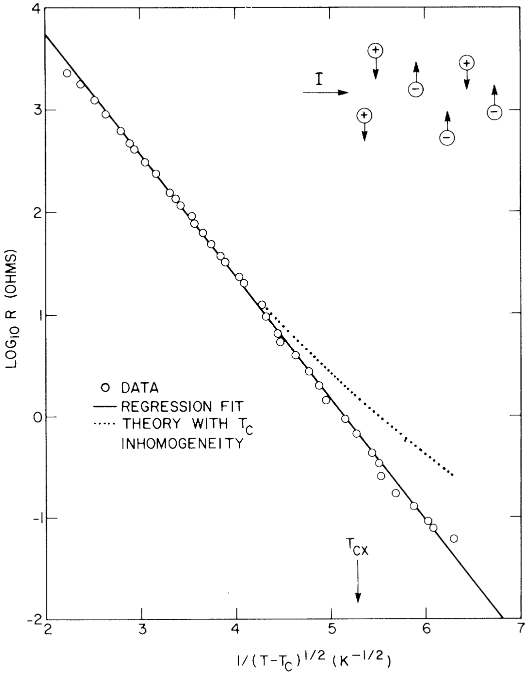
Resistivity is given by vortex density:

$$V \sim J^{1+2\frac{T_{BKT}}{T}}$$



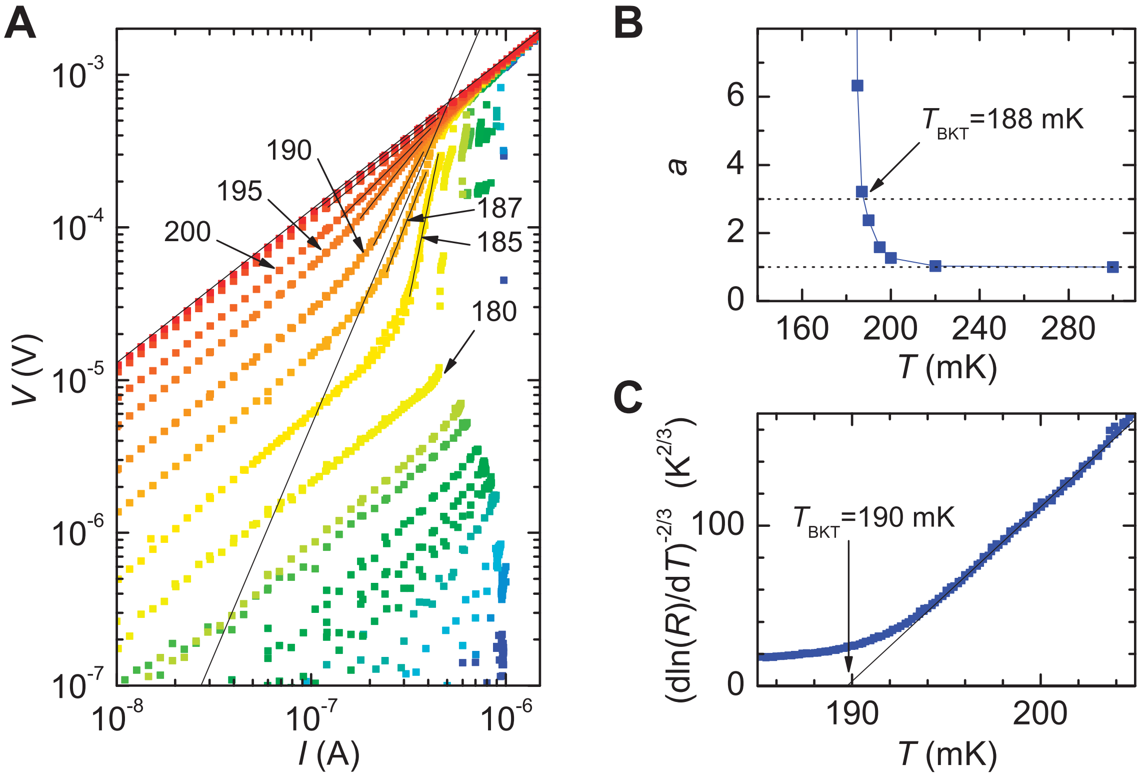
Experimental observation: InO_x





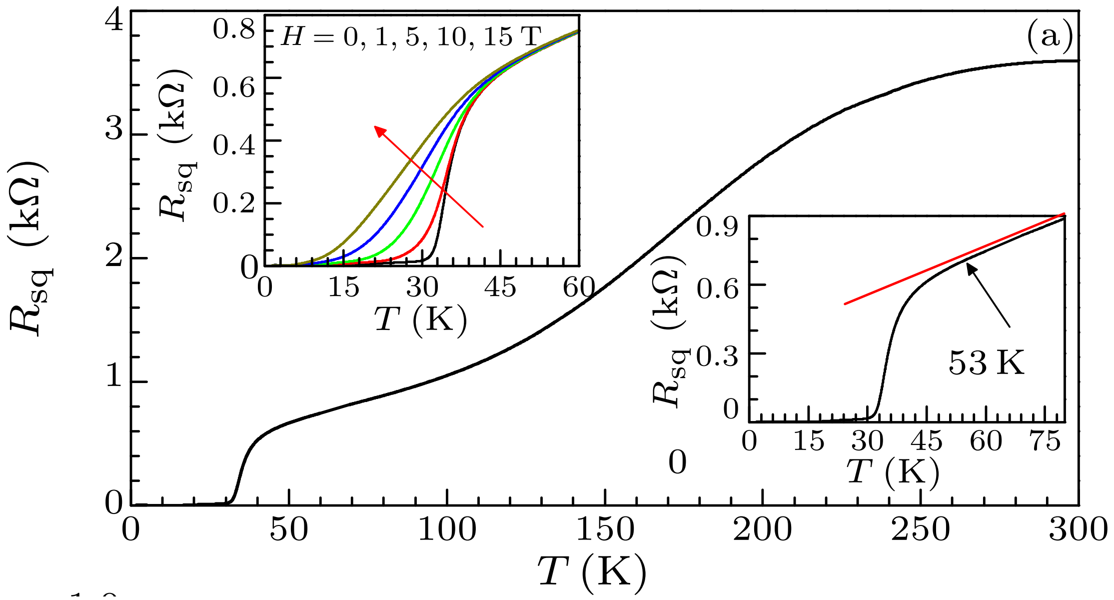
Ref: Hebard, Fiory PRL 1983

Experimental observation: LAO/STO



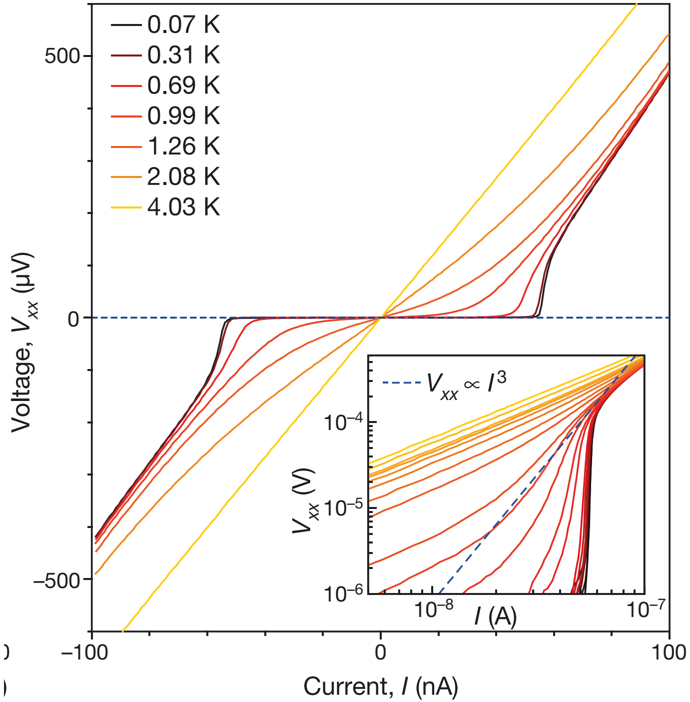
Ref: Reyren et al, Science 2007

Experimental observation: FeSe/STO??

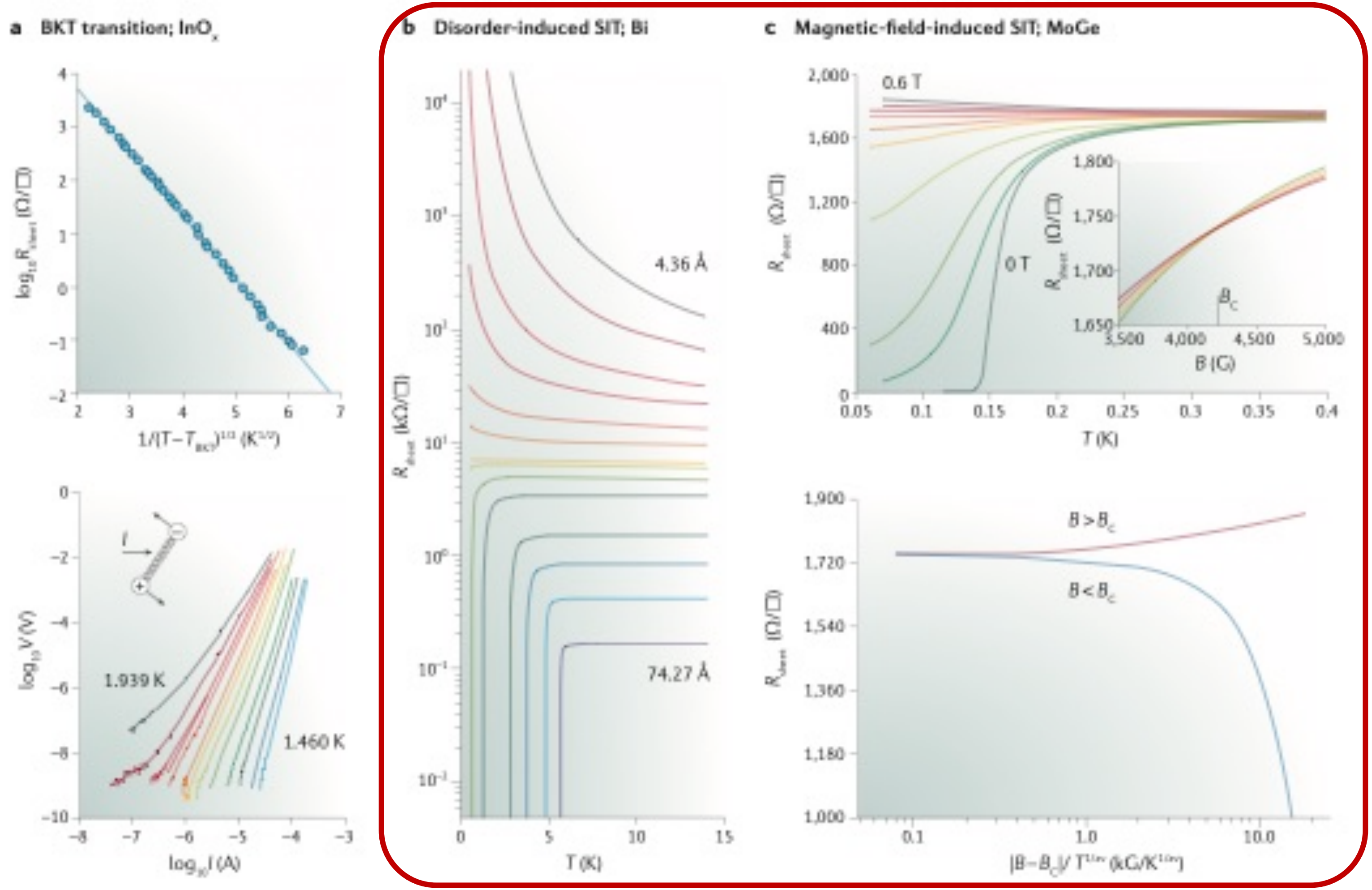


Ref: Wang et al., CPL 2012

Experimental observation: Twisted bilayer graphene



Ref: Cao et al., Nature 2018



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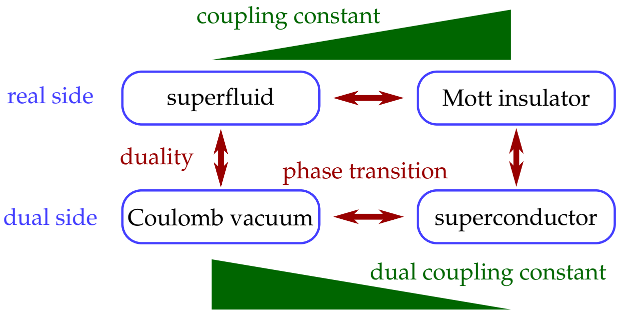
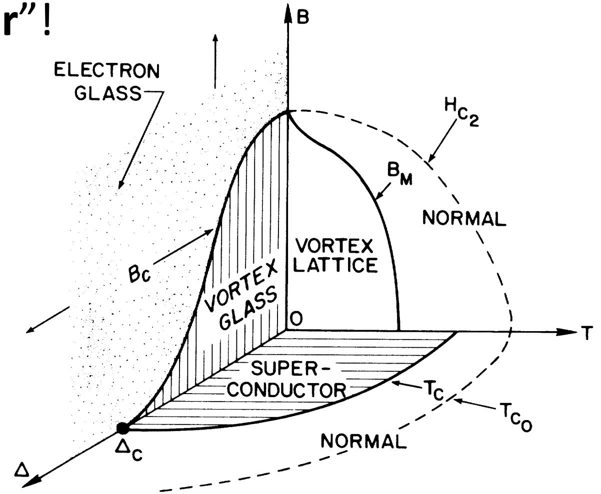
Vortex-particle duality

The vortices are **“charged” particles** with a logarithmic interaction

These are bosons that can form a “**superconductor**”!





Two transitions with condensation of vortices:

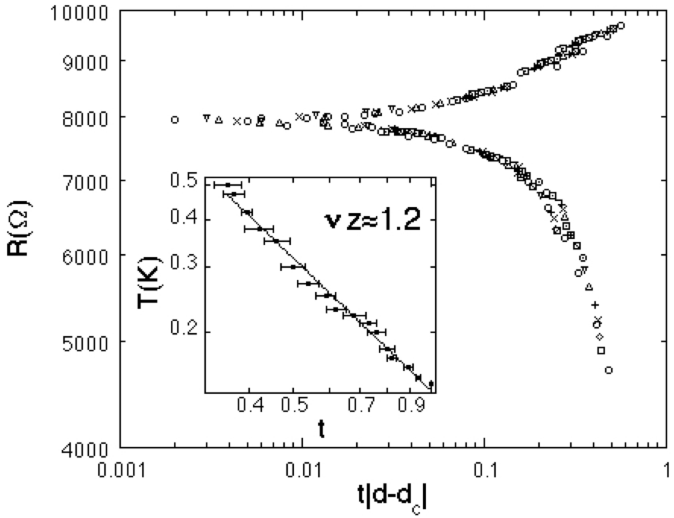
- Disorder-driven SIT
- Magnetic-field tuned SIT

Ref: Fisher 1990

Superconductor-Insulator transition

So SIT is **Quantum Phase Transition** with control parameter δ

Diverging correlation length $\xi \sim |\delta|^{-\nu}$ and time $\xi_\tau \sim \xi^z$



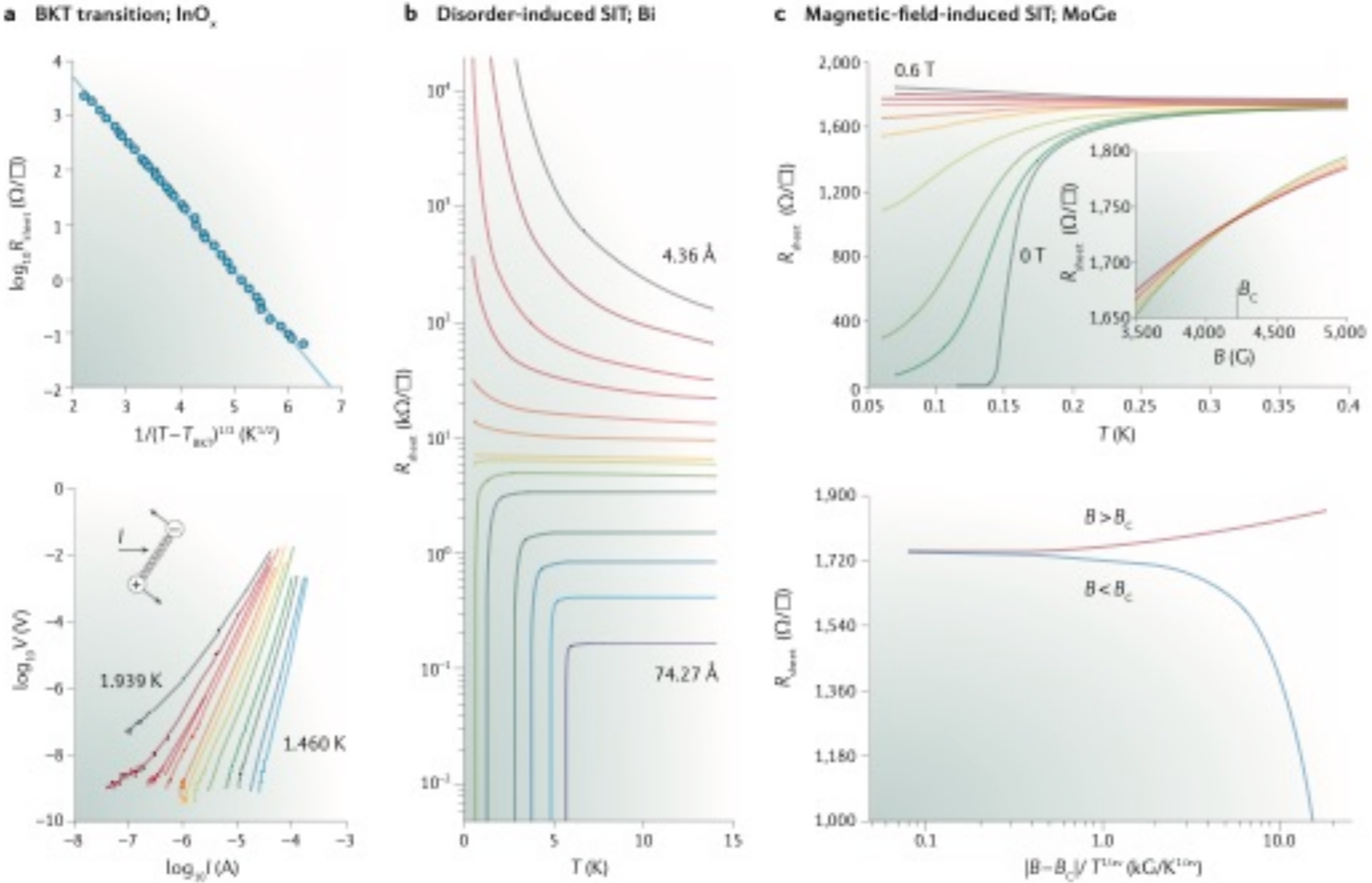
Scaling form for resistivity

$$R = R_0 \; f(\delta T^{-1/\nu z})$$

At the transition: **universal resistivity**

$$R_c = \frac{h}{4e^2} = 6.5 \text{ k}\Omega$$

Ref: Goldman 2010



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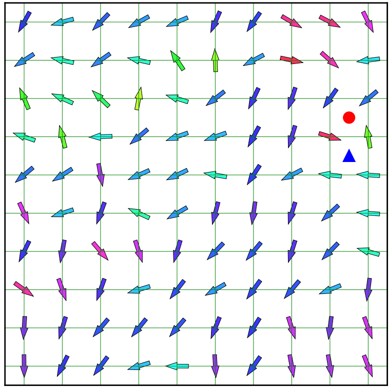
Summary: 2D superconductors



Described by **XY model**

No true long-range order

Resistivity governed by **vortices**
(BKT transition, SIT transition)

Open Questions:

- What happens to the **Cooper pairs** above a BKT transition?
- Role of vortices in **unconventional** order parameters (d-wave, topological)?
- Nature of the **Quantum metallic state**?
- What is the **glue**?
- Role of vortex **pinning**?